

## MATH 1700: TEST 02 (100 POINTS)

NAME: \_\_\_\_\_

**DIRECTIONS:** Make sure your work is neat and complete and uses the techniques demonstrated in class.

1. Verify the following identities.

(a)  $\frac{\cos(\theta)}{\sin^2(\theta)} = \csc(\theta) \cot(\theta)$

(b)  $\frac{1}{1 + \sin(\theta)} = \sec^2(\theta) - \sec(\theta) \tan(\theta)$

2. Suppose  $\frac{3\pi}{2} < \theta < 2\pi$  with  $\tan(\theta) = -\frac{5}{12}$ .

(a) Find and simplify  $\cos(\theta)$  and  $\sin(\theta)$ .

(b) Find and simplify:  $\sin(2\theta)$  and  $\cos(2\theta)$ .

(c) In which quadrant does the terminal side of  $2\theta$  lie? Explain your reasoning.

3. Find  $A$ ,  $\omega$ ,  $\phi$ , and  $B$  so that  $\cos(3t) - \sqrt{3} \sin(3t) = A \sin(\omega t + \phi) + B$ .

4. Use the so-called 'power reduction' identities:

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2} \quad \cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

to help you verify:  $8 \sin^4(\theta) = \cos(4\theta) - 4 \cos(2\theta) + 3$

**HINT:**  $\sin^4(\theta) = \sin^2(\theta) \sin^2(\theta) \dots$

5. Find the exact values of the following:

(a)  $\arccos\left(\frac{\sqrt{3}}{2}\right)$

(f)  $\operatorname{arccsc}(2)$

(b)  $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

(g)  $\cos\left(\arccos\left(-\frac{3}{5}\right)\right)$

(c)  $\operatorname{arccot}(-1)$

(h)  $\arccos\left(\cos\left(\frac{7\pi}{6}\right)\right)$

(d)  $\arctan(-1)$

(i)  $\arcsin\left(\sin\left(\frac{3\pi}{2}\right)\right)$

(e)  $\operatorname{arcsec}(-2)$

(j)  $\tan(\operatorname{arccot}(5))$

6. Let  $\theta = \arctan(-2)$ .

(a) Sketch  $\theta$  in standard position. Find the coordinates  $(x, y)$  of a point on the terminal side of  $\theta$ .

(b) Find the exact value of  $\sin(\theta)$ .

7. (a) Use identities to help you find the exact value of the following:  $\sin\left(2\arccos\left(\frac{7}{25}\right)\right)$

(b) Use identities to help rewrite as a function of  $x$ :  $\sin(2\arccos(x))$

8. Find the solutions to the following equations which lie in  $[0, 2\pi)$ . Exact answers only, please.

(a)  $\cos(3\theta) = -1$

(b)  $\tan(\theta) + 117 = 0$

(c)  $\tan^2(t) = \sec(t) + 1$

(d)  $\cos(2t) = 5\sin(t) - 2$

**BONUS:** Find exact values of  $A$ ,  $\omega$ , and  $\phi$  so that  $5\cos(\pi t) - 12\sin(\pi t) = A\sin(\omega t + \phi)$ .

# IMPORTANT POINTS ON THE UNIT CIRCLE

