MATH 2850: TEST 07 (25 points.)

NAME: ____________________

DUE: Wednesday, March 6th, at the beginning of class.

DIRECTIONS: Show all work.

1. Suppose for the ODE: $a\,y'' + b\,y' + c\,y = f(x)$, the complementary solution is: $y_c = c_1\,e^{-x} + c_2\,e^{3x}$.

   Write the *form* for the particular solution, y_p in each of the cases below.[1]

   (a) $f(x) = 3x^2$

   (b) $f(x) = \sin(4x) - e^{4x}$

   (c) $f(x) = e^{-x}\cos(x)$

   (d) $f(x) = x^2\,e^{-3x}$

   (e) $f(x) = x^2\,e^{3x}$

$\overline{10}$

[1] **NOTE:** You do not need to solve for the coefficients!

2. Consider the ODE: $y'' + 2y' + y = f(x)$.

(a) Find the complementary solution, y_c.

(b) Write the *form* of the particular solution, y_p in the following cases.[2]

- $f(x) = x - e^{3x}$

- $f(x) = \sin(3x)$

- $f(x) = e^{-x}\cos(x)$

- $f(x) = 3x^2 e^{-x}$

$\overline{10}$

[2] **NOTE:** You do not need to solve for the coefficients!

3. (a) Use the definition of linear independence to show $\left\{x^4, |x|x^3\right\}$ is linearly independent on $(-\infty, \infty)$.

(b) Show the Wronskian $W\left(x^4, |x|x^3\right) = 0$ for all x.

HINT: It may help to write $|x|x^3 = \begin{cases} -x^4, & \text{if } x < 0 \\ x^4, & \text{if } x \geq 0 \end{cases}$