MATH 2850: TAKE HOME 14 (25 points.)

NAME: ______________________________

DUE: Wednesday, May 1st, at the beginning of class.

DIRECTIONS: Show all work.

1. Find: $\int_0^t u\cos(t-u)\,du$ using Laplace and Inverse Laplace Transforms.

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2. It turns out that if f is periodic of period T, that is, $f(t+T) = f(t)$ for all t in the domain of f, then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T f(t)e^{-st}\, dt$$

(a) Find $\mathcal{L}\{\sin(t)\}$ using this formula and show it reduces to the formula we have been using in class.[1]

(b) Find $\mathcal{L}\{f(t)\}$ where $f(t)$ is the 'sawtooth wave' $f(t) = t - n$ on $[n, n+1)$ for $n = 0, 1, 2, \ldots$[2]

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[1]Feel free to use the formula: $\displaystyle\int e^{at}\sin(bt) = \frac{e^{at}}{a^2+b^2}\left(a\sin(bt) - b\cos(bt)\right)$

[2]That is, $f(t)$ is the line segment $f(t) = t$, $0 \le t < 1$ copied and pasted down along the t axis.

3. Prove the property used in number 2 by working through the followings steps:

- explain why $\int_0^\infty f(t)e^{-st}\,dt = \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} f(t)e^{-st}\,dt$
- make a change of variables $u = t - nT$ (so $t = u + nT$) and use the fact that f has period T to get:

$$\int_{nT}^{(n+1)T} f(t)e^{-st}\,dt = \int_0^T f(u+nT)\,e^{-su-snT}\,du = e^{-snT}\int_0^T f(u)e^{-su}\,du$$

- use the Geometric Series Formula: $\sum_{n=0}^{\infty} e^{-snT} = \sum_{n=0}^{\infty}\left(e^{-sT}\right)^n = \dfrac{1}{1-e^{-sT}}$

NOTE: Make any 'reasonable' assumptions you need.