MATH 2850: TAKE HOME 13 (25 points.)

NAME: _____

DUE: Wednesday, April 24th, at the beginning of class.

DIRECTIONS: Show all work.

1. Solve: $y'' + 2y' + y = f(t)$, $y(0) = y'(0) = 1$, where $f(t) = \begin{cases} 4e^t & \text{if } 0 \leq t < 1 \\ 0 & \text{if } t \geq 1 \end{cases}$.

   HINT: Recall $e^{t+1} = e^t e^1 = e\, e^t$ ...

2. Suppose $\mathcal{L}\{f(t)\} = F(s)$.

(a) Show $\mathcal{L}\{t\,f(t)\} = -\dfrac{d}{ds}F(s)$ by starting with the integral definition of $F(s)$ and finding $\dfrac{d}{ds}F(s)$.

HINT: Under suitable conditions: $\dfrac{d}{ds}\displaystyle\int_0^\infty g(s,t)\,dt = \int_0^\infty \frac{\partial}{\partial s}g(s,t)\,dt$.

(b) Find $\mathcal{L}\left\{te^{3t}\right\}$ two ways:

i. Using the Forward Shift Theorem

ii. Using the result you proved in part (a).

(c) **BONUS:** Applying the result in part (a) repeatedly, find formulas for:

i. $\mathcal{L}\left\{t^2 f(t)\right\}$

ii. $\mathcal{L}\left\{t^3 f(t)\right\}$

iii. Generalize a formula for $\mathcal{L}\left\{t^k f(t)\right\}$ for $k = 1, 2, 3, \ldots$.

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