MATH 2850: TAKE HOME 09 (25 points.)

NAME: ______________________________

DUE: Wednesday, March 27th, at the beginning of class.

DIRECTIONS: Show all work.

1. A 2 kilogram mass stretches a spring 2.45 meters. Suppose the mass is released 3 meters above the equilibrium position with a downward velocity of 6 meters per second.

   (a) Write the corresponding IVP which corresponds to this situation. Be careful with units!

   NOTE: Use $g = 9.8\text{m/s}^2$ as the acceleration due to gravity.

   (b) Solve your IVP from part (a).

   Write your solution in the form $y = c_1 \sin(\omega t) + c_2 \cos(\omega t)$ as well as $y = A \sin(\omega t + \phi)$.

   What is the period and (ordinary) frequency of the motion?

$\overline{10}$

2. The same method we employ to solve Cauchy-Euler equations can help us find explicit formulas for the terms of sequences defined recursively. Consider the famous Fibonacci sequence F_n described described by two initial terms F_0 and F_1 along with a recurrence relation:

$$F_0 = 1, \qquad F_1 = 1, \qquad F_{n+2} = F_{n+1} + F_n, \qquad \text{for } n \geq 0$$

For example, for $n = 0$, $F_2 = F_1 + F_0 = 1 + 1 = 2$ and for $n = 1$, we get $F_3 = F_2 + F_1 = 2 + 1 = 3$.

(a) Use the recurrence relation to find F_4 and F_5.

(b) Suppose we assume $F_n = x^n$ for some real number x so that $F_{n+1} = x^{n+1}$ and $F_{n+2} = x^{n+2}$.

Substitute these terms into the recurrence relation to solve for x.

HINT: You should get two solutions. Let's call them $\phi = \dfrac{1+\sqrt{5}}{2}$ and $\psi = \dfrac{1-\sqrt{5}}{2}$.

(c) Let $F_n = c_1\,\phi^n + c_2\,\psi^n$. Find the values of c_1 and c_2 so that $F_0 = 1$ and $F_1 = 1$.

Feel free to leave your answers in terms of ϕ and ψ.

NOTE: '$F_0 = 1$ and $F_1 = 1$' are analogous to initial conditions.

(d) Check your answer to part (c) by simplifying the formula for F_0 and F_1 to show both are 1.

(e) With the help of a calculator or graphing utility, use your formula from part (c) to find F_5 and F_{42}.

(f) Use your answer to part (c) to show: $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \phi$.

HINT: First show that $-1 < \frac{\psi}{\phi} < 1$ so that $\lim_{n\to\infty} \frac{\psi^n}{\phi^n} = \lim_{n\to\infty} \left(\frac{\psi}{\phi}\right)^n = 0$.

Then factor out ϕ^{n+1} from F_{n+1} and ϕ^n from F_n to help you work through the limit.

NOTE: The number 'ϕ' is often called the 'Golden Ratio.' The fact that ϕ is the limit of the ratio of successive Fibonacci Numbers has some interesting implications in nature ...