MATH 2850: TAKE HOME 01 (25 points.)

NAME: ____________________

DUE: Wednesday, January 24th, at the beginning of class.

DIRECTIONS: Show all work.

1. Verify $y = e^{x^2} \int_2^x e^{-t^2}\, dt + 4e^{x^2-4}$ is a solution to the IVP: $y' = 2xy + 1$, $y(2) = 4$.

   HINT: Recall: $D_x \left[\int_a^x f(t)\, dt \right] = f(x)$. (i.e., differentiation undoes integration.)

   Also don't forget the product rule!

2. (a) Verify $y = \dfrac{5}{1 - C\,e^{5x}}$ is a one-parameter family of solutions to the DE: $\dfrac{dy}{dx} = y\,(y - 5)$.

(b) Show $y = 0$ is a solution to $\dfrac{dy}{dx} = y\,(y - 5)$.

(c) Based on your answer to part (b), is $y = \dfrac{5}{1 - C\,e^{5x}}$ the general solution to $\dfrac{dy}{dx} = y\,(y - 5)$? Explain.

3. Consider the DE: $\dfrac{dy}{dx} = \dfrac{y^3 - 1}{xy^2}$.

(a) Use implicit differentiation to show $C\,x^3 + y^3 = 1$ is a family of solutions to the DE.

(b) Use implicit differentiation to show $x^3 + C\,y^3 = C$ is a family of solutions to the DE.

(c) If we impose the IC $y(2) = 1$ to the DE above, which of the families in (a) or (b) yield a solution?

What is a solution?

4. Verify $y = c_1 \cosh(kx) + c_2 \sinh(kx)$ is a two parameter family of solutions to $y'' - k^2 y = 0$.

5. Consider the DE: $\dfrac{dy}{dx} = \sin(x) + \sin(y)$.

   (a) Using Slope Field Plotter , plot the solutions to this DE containing $A = (0, 0)$ and $B = (0, 2\pi)$.

   NOTE: Remember you can adjust the values for y-min and y-max, increase the density of the lineal elements, and zoom out, as needed. A good window for this problem is $[-10, 10] \times [-10, 10]$ or larger.

   What appears to be the geometric relationship between these two solutions? Include a screenshot.

   Use the DE (and some trigonometry!) to explain your observation.

   (b) Plot the solution to the DE which contains $C = (0, 1)$ and slowly drag point C to point B.

   What happens?

   NOTE: Exploring this type of behavior would be a great semester project!