MATH 1650: TAKE HOME 02 - 20 POINTS

NAME: ____

DIRECTIONS: To receive full credit, make sure your work is neat and complete.

SECTION 2.1 PRACTICE PROBLEMS

1. Let $f(x) = (2x+1)^2(x-3)$.

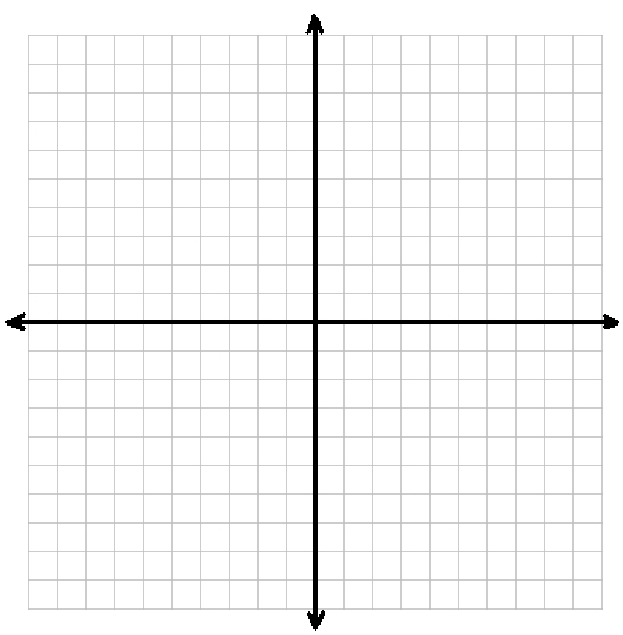
   (a) Find the leading term of $f(x)$ and use this to describe the end behavior of the graph of $y = f(x)$.

   - leading term: __________
   - as $x \to -\infty$, $f(x) \to$ ____
   - as $x \to \infty$, $f(x) \to$ ____

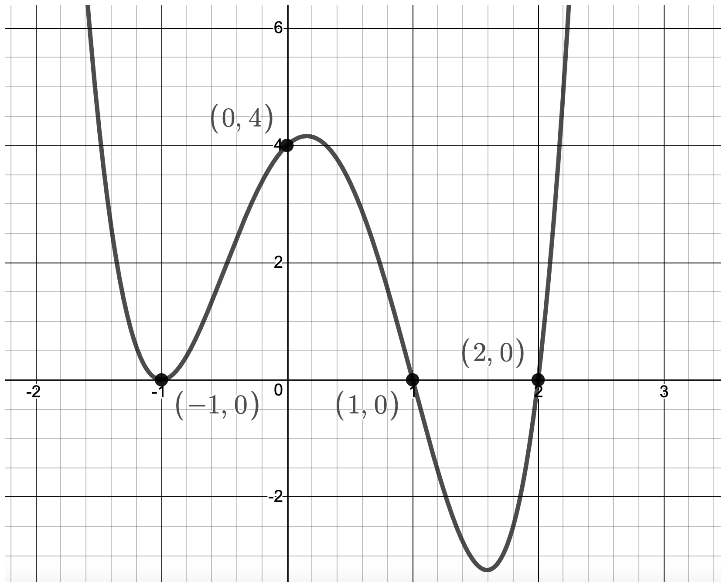
   (b) Find the real zeros of f and state the multiplicity of each.

   (c) Find the y-intercept of the graph of $y = f(x)$.

   (d) Graph $y = f(x)$ in the space provided. Label all the intercepts.



2. Below is the complete graph of a polynomial function, p



$y = p(x)$

(a) Is the degree of $p(x)$ even or odd? Explain.

(b) Is the leading coefficient of $p(x)$ positive or negative? Explain.

(c) Find the real zeros of p and explain whether the multiplicity of each is even or odd.

(d) Find $p(0)$ and solve $p(x) \geq 0$.

- $p(0) =$ ___
- $p(x) \geq 0$ on ______________________________

3. According to the US Census, Table A-3, the share of money income in 2020 is given in the table below on the left. From these data, we can create a cumulative distribution, $y = L(x)$ called the **Lorenz Curve**.

The number $L(x)$ gives the percentage of the total national income earned by the bottom x percent of wage earners, ranked from lowest income to highest income. Since the population here is separated into 'quintiles,' each data point corresponds to 20% of the population. So, for example, $L(20)$ is the percentage of money income earned by the lowest 20% of wage earners. In this case, we see $L(20) = 3.0$. The number $L(40)$ is the percentage of the money income earned by the bottom 40% of wage earners - so this includes not only the money from the Second Quintile, but also the Lowest Quintile: $L(40) = 8.1 + L(20) = 8.1 + 3.0 = 11.1$. Likewise, $L(60)$ is the total income share of the bottom 60% of wage earners which includes the income from the Middle, Second, and Lowest Quintiles: $L(60) = 14.0 + L(40) = 14.3 + (8.1 + 3.0) = 25.1$.

Continuing in this manner, we get $L(80) = 47.7$ and $L(100) = 99.9$, which is what we would expect since 100% of the income (well rounded up) is earned by 100% of the population.

Portion of Population	Percent of Money Income	percent wage earners, x	percent income, $L(x)$
Lowest Quintile	3.0	20	3.0
Second Quintile	8.1	40	11.1
Middle Quintile	14.0	60	25.1
Fourth Quintile	22.6	80	47.7
Highest Quintile	52.2	100	99.9

(a) Use Desmos to make a table for the pairs $(x, L(x))$.

Find a fourth degree polynomial which fits these data.

(b) Find and interpret $L(90)$.

(c) Use $L(x)$ to approximate the percentage of income earned by the top 5 % of the wage earners.

(d) Based on your model, what is $L(0)$? Does this make sense? Explain.

SECTION 2.2 PRACTICE PROBLEMS

1. Use synthetic division to perform the following polynomial divisions. Identify the quotient and remainder.

   Check that $p(x) = d(x)q(x) + r(x)$.

   (a) $\left(z^3 + 8\right) \div (z + 2)$

   (b) $\left(3t^3 - t + 4\right) \div \left(t - \frac{2}{3}\right)$

   (c) $\left(x^6 - 6x^4 + 12x^2 - 8\right) \div \left(x + \sqrt{2}\right)$

2. For each of the following, you are given a polynomial function and one of its zeros.

   Find the remaining real zeros and factor the polynomial.

   (a) $x^3 - 24x^2 + 192x - 512, \quad c = 8$

   (b) $2t^3 - 3t^2 - 11t + 6, \quad c = \frac{1}{2}$

(c) $t^5 + 2t^4 - 12t^3 - 38t^2 - 37t - 12$, $c = -1$ is a zero of multiplicity 3

3. **EXPLORATION:** Suppose a is a nonzero real number. Find the quotients below using synthetic division.

- $\dfrac{x-a}{x-a}$
- $\dfrac{x^2-a^2}{x-a}$
- $\dfrac{x^3-a^3}{x-a}$
- $\dfrac{x^4-a^4}{x-a}$
- $\dfrac{x^5-a^5}{x-a}$

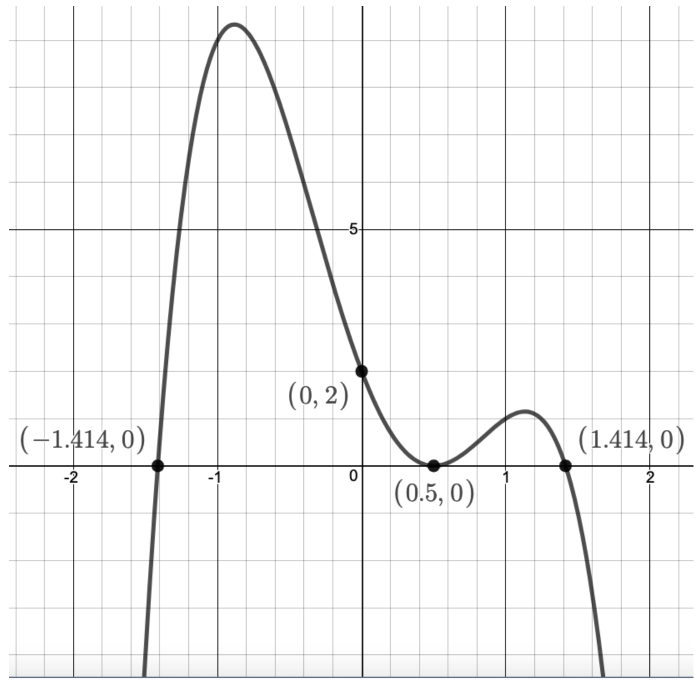
Based on the pattern that evolves, find the quotient: $\dfrac{x^{10}-a^{10}}{x-a}$. What about $\dfrac{x^n-a^n}{x-a}$?

SECTION 2.3 PRACTICE PROBLEMS

1. Let $p(x) = -4x^4 + 4x^3 + 7x^2 - 8x + 2$.

   Use the graph of $y = p(x)$ and synthetic division to find the exact values of all the real zeros of p.

   NOTE: the graph **suggests** the value of zeros; synthetic division which **proves** the value of the zeros.



$y = p(x)$

2. The function $P(x) = -5x^3 + 35x^2 - 45x - 25$, for $x \geq 0$, gives the profit in **thousands** of dollars obtained by making and selling x **hundred** 'Slug Mugs.'[1]

Use Desmos to help you make a Sign Diagram for $P(x)$ and use it to solve $P(x) > 0$.

Round your answers to two decimal places. **HINT:** Don't forget that $x \geq 0$.

What does your solution mean in terms of money and mugs?

[1]Inspired by the emotes of the amazing twitch streamer sluggorider.

SECTION 2.4 PRACTICE PROBLEMS

1. (a) Find all complex (real and nonreal) zeros of $f(x) = 2x^3 - 5x^2 + 8x - 20$

   HINT: $f(x)$ factors by grouping!

   (b) Factor $f(x)$ over the complex numbers.

2. Let $p(x) = 4x^4 - 16x^3 + 13x^2 - 36x + 9$.

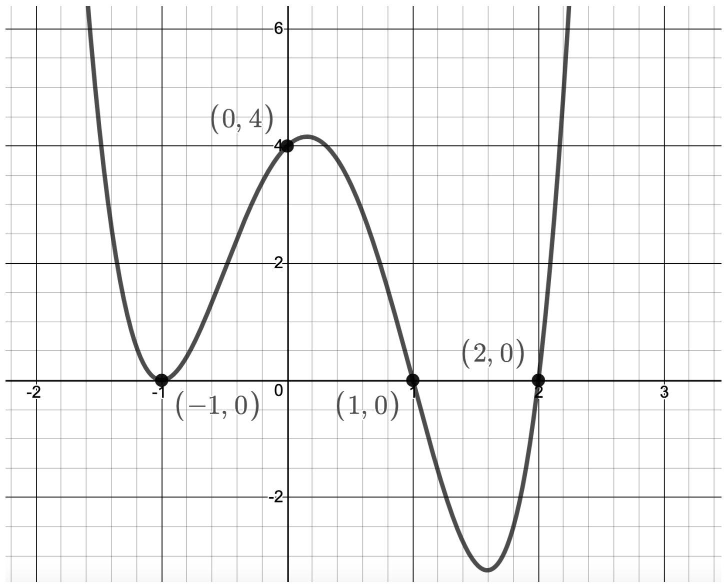
   (a) Show $x = \dfrac{3i}{2}$ is a zero of p and find the remaining complex (real and nonreal) zeros of p.

   (b) Factor $p(x)$ over the complex numbers.

   (c) Factor $p(x)$ over the real numbers.

3. The complete graph of the polynomial function $y = p(x)$ is below.

NOTE: This is the same polynomial from Section 2.1's Practice Problems!



$y = p(x)$

Find a possible formula for $p(x)$. Explain your reasoning!

4. Write the factored form of a **fifth** degree polynomial function $p(x)$ with real number coefficients which satisfies all of the following characteristics:

   - As $x \to -\infty$, $p(x) \to \infty$
   - As $x \to \infty$, $p(x) \to -\infty$
   - the graph touches and rebounds from the $x-$axis at $(-1, 0)$
   - $x = 2 - i$ is a zero
   - the y-intercept of the graph of $y = p(x)$ is also an x-intercept.

   NOTE: There are several (infinitely many!) answers!

SECTION 3.1 PRACTICE PROBLEMS

1. Let $r(x) = \dfrac{2x^2 + x - 1}{x^2 - 1}$.

   (a) Algebraically find the values excluded from the domain of r and explain why they are excluded.

   (b) Reduce $r(x)$ to lowest terms.

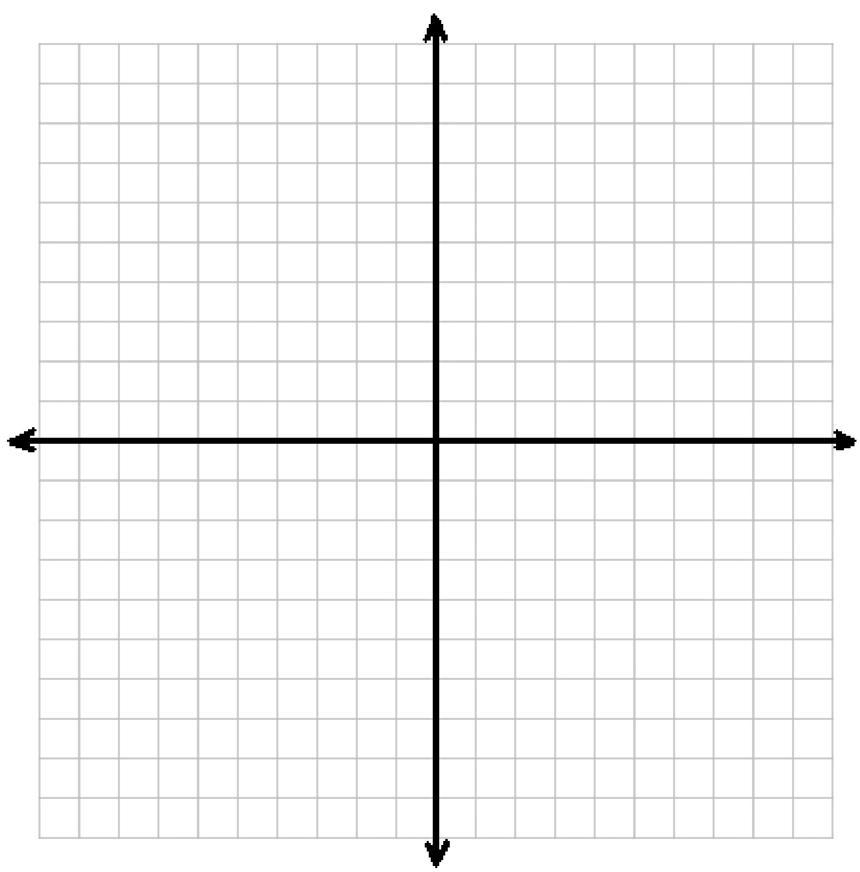
   (c) Find the x and y intercepts of the graph of $y = r(x)$.

(d) Algebraically find the vertical asymptote and hole in the graph of $y = r(x)$.

vertical asymptote: hole:

(e) Explain how you know the graph of $y = r(x)$ has a horizontal asymptote and find it.

(f) Sketch an incredibly detailed graph of $y = r(x)$ on the axes provided.



2. Let $g(x) = \dfrac{2x^2 + 5x + 4}{x + 1}$

(a) Algebraically show the graph of $y = g(x)$ has no x-intercepts.

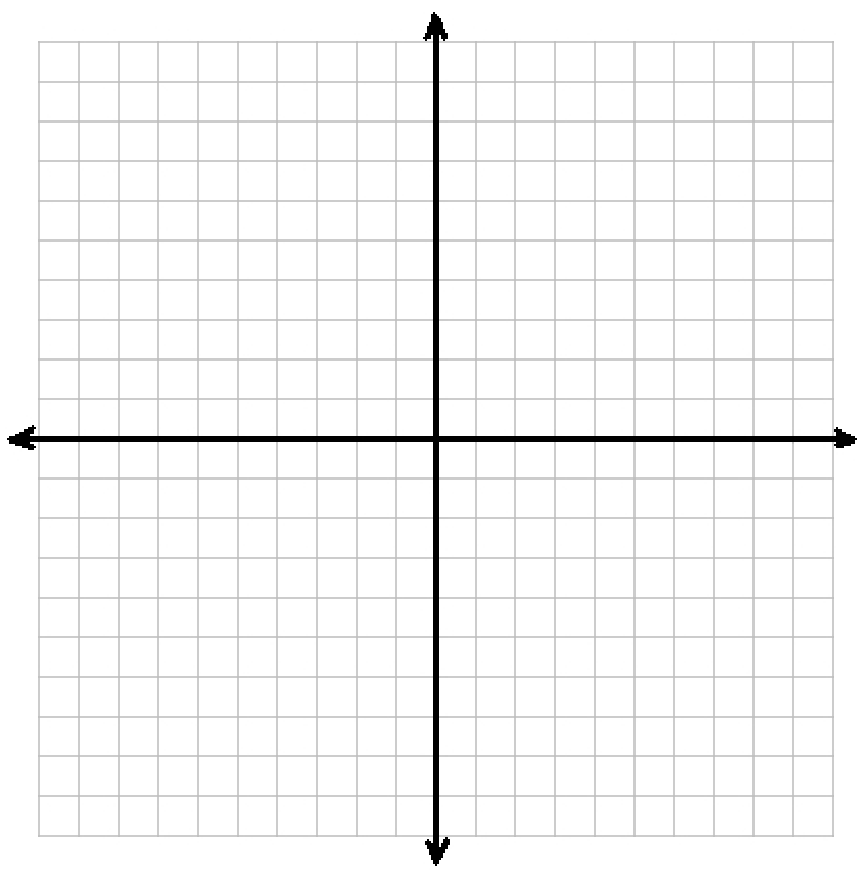
(b) Find the y-intercept(s) to the graph of $y = g(x)$.

(c) Algebraically find the vertical asymptote to the graph of $y = g(x)$.

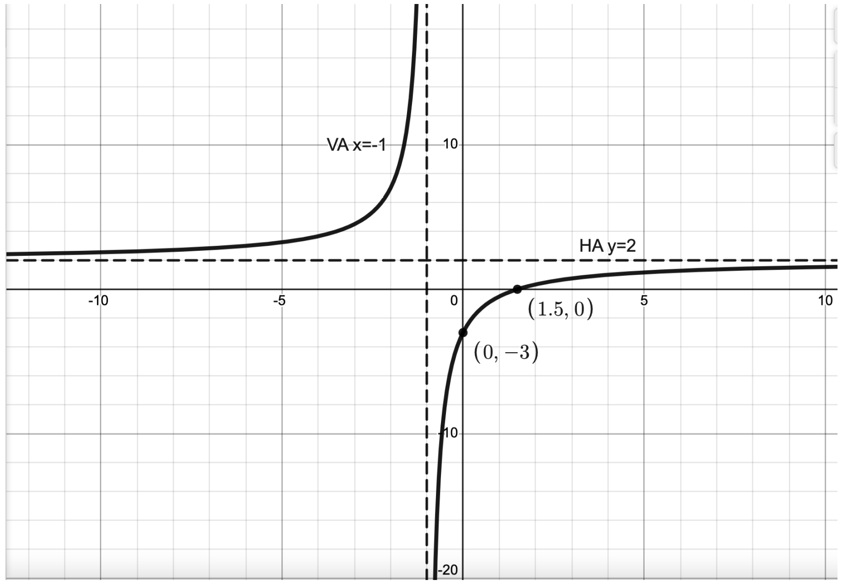
(d) How do you know there are no holes in the graph of $y = g(x)$?

(e) Explain how you know the graph of $y = g(x)$ has a slant asymptote and find it.

(f) Sketch an incredibly detailed graph of $y = g(x)$ on the axes provided.

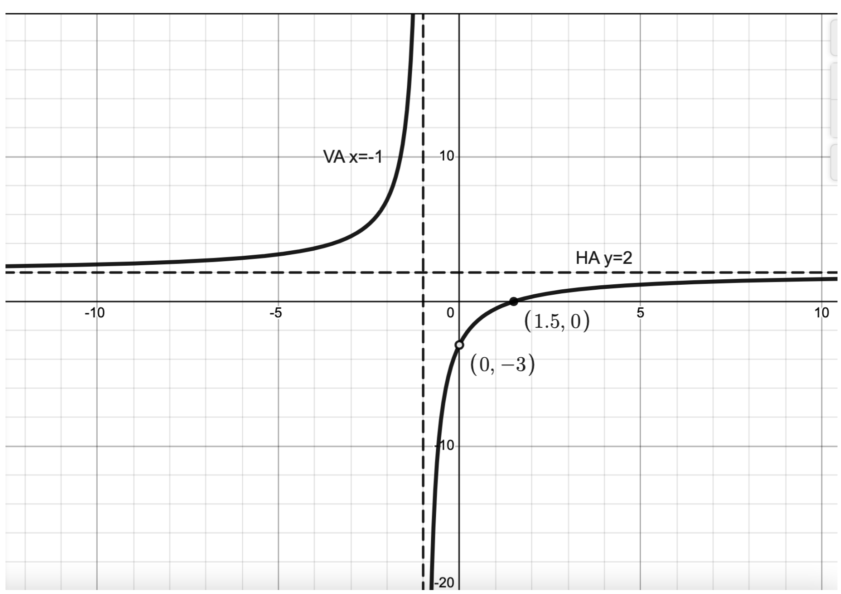


3. (a) Find a possible formula for $f(x)$ whose graph is below. Explain your reasoning!



$y = f(x)$

(b) Modify your answer to part (a) to find a possible formula for $g(x)$ whose graph is below. Explain your reasoning!



$y = g(x)$

SECTION 3.2 PRACTICE PROBLEMS

1. Let $r(x) = \dfrac{x^2 - x - 12}{x^2 + x - 6}$.

   (a) Algebraically find the values excluded from the domain of r and explain why they are excluded.

   (b) Reduce $r(x)$ to lowest terms.

   (c) Find the x and y intercepts of the graph of $y = r(x)$.

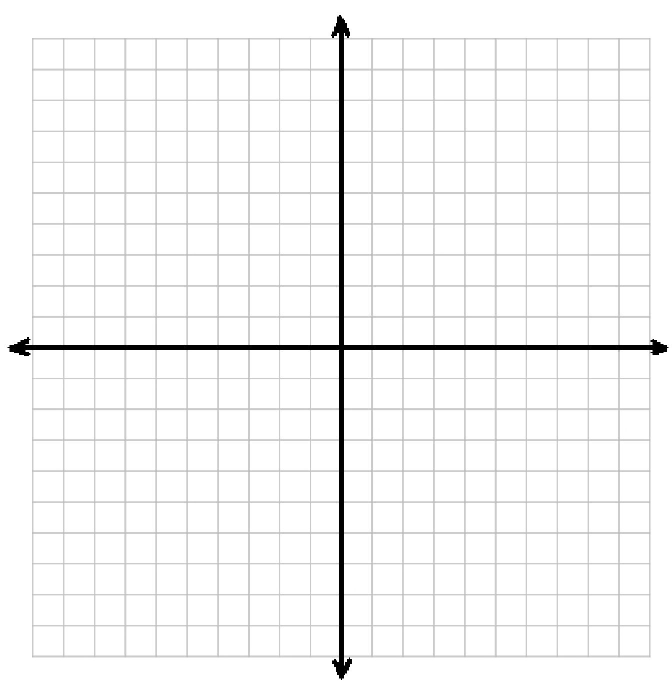
(d) Algebraically find the vertical asymptote and hole in the graph of $y = r(x)$.

vertical asymptote: hole:

(e) Explain how you know the graph of $y = r(x)$ has a horizontal asymptote and find it.

(f) Make a Sign Diagram for r.

(g) Sketch an incredibly detailed graph of $y = r(x)$ on the axes provided.



2. Let $r(x) = \dfrac{x^3 + 2x^2 + x}{x^2 - x - 2}$.

(a) Algebraically find the values excluded from the domain of r and explain why they are excluded.

(b) Reduce $r(x)$ to lowest terms.

(c) Find the x and y intercepts of the graph of $y = r(x)$.

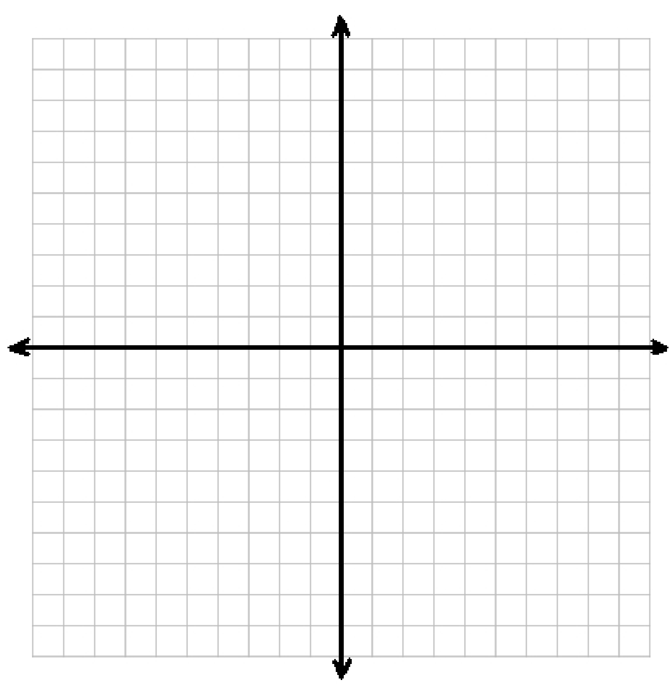
(d) Algebraically find the vertical asymptote and hole in the graph of $y = r(x)$.

vertical asymptote: hole:

(e) Explain how you know the graph of $y = r(x)$ has a slant asymptote and find it.

(f) Make a Sign Diagram for r.

(g) Sketch an incredibly detailed graph of $y = r(x)$ on the axes provided.



SECTION 3.3 PRACTICE PROBLEMS

3. Solve using a Sign Diagram: $\dfrac{2x-3}{x+2} \leq 4$. Write your answer using interval notation.

4. **EXPLORATION:** Skippy solves the inequality:

$$\frac{x-3}{x^2+1} \geq 0$$

by multiplying both sides of the inequality by (x^2+1) to get $x-3 \geq 0$. He then arrives at the correct answer of $x \geq 3$. He tries this same technique with the inequality:

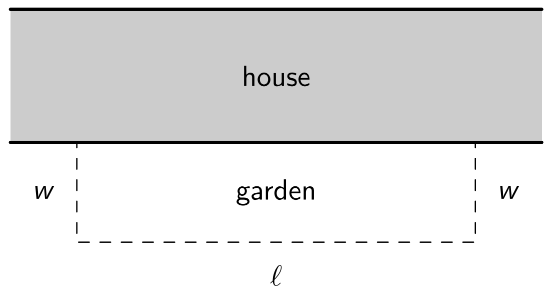
$$\frac{x-3}{x^2-1} \geq 0.$$

That is, he multiplies both sides of the inequality by (x^2-1), gets $x-3 \geq 0$, and so arrives at the same answer, $x \geq 3$. Unfortunately $x \geq 3$ is not the correct answer here.

(a) Why didn't the same technique work for both problems? In your response, be sure to explain why Skippy's technique worked for the first problem but not the second problem.

(b) What technique would work correctly for both problems?

5. Taylor wants to plant a 450 square foot rectangular garden along the side of her home. Since the garden will be against the house, she has no need to fence along that side of the garden.



(a) Find a formula for the amount of fencing required F in terms of w and ℓ.

(b) Use the fact that the area of the garden is to be 450 square feet to help you find a formula for the amount of fencing required F as a function of just the width, w.

(c) Use desmos to graph F to determine the minimum amount of fencing needed and the dimensions of the garden to achieve that minimum.

CHAPTER 4 PRACTICE PROBLEMS

1. Find the domain of the following functions. Write your answers using interval notation.

   (a) $f(x) = \sqrt{3x+1}$

   (b) $g(t) = \dfrac{\sqrt[3]{2t-1}}{t^2+1}$

   (c) $F(x) = \sqrt[4]{\dfrac{2x+1}{x-3}}$

   (d) $G(t) = \dfrac{t^2+1}{1-\sqrt{t+3}}$

2. Find the domain of the following functions. Write your answers using interval notation.

(a) $f(x) = (2x-3)^{-\frac{2}{3}}$

(b) $F(x) = (2x-3)^{-\frac{3}{2}}$

(c) $g(t) = \dfrac{t^{0.5}}{4 - t^{0.25}}$

(d) $G(t) = \dfrac{t^{0.58}}{4 - t^{0.24}}$

3. Use a Sign Diagram to solve: $(x-1)^{\frac{2}{5}} \leq (x-1)^{-\frac{3}{5}}$

4. Let $f(x) = \dfrac{2x+1}{3-x}$.

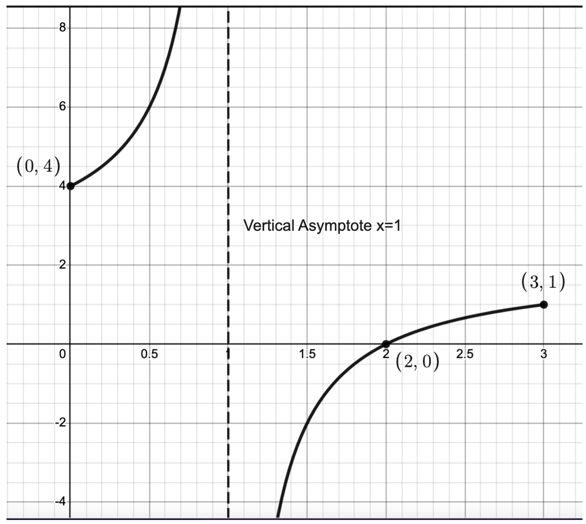
(a) Make a Sign Diagram for $f(x)$

(b) Use part (a) to help you find the domain of $g(x) = \sqrt{f(x)}$

(c) Use part (a) to help you find the domain of $h(x) = [f(x)]^{-3/2}$

(d) Use part (a) to help you find the domain of $T(x) = [f(x)]^{-2/3}$

5. Below is the complete graph of $y = f(x)$. Note the domain here is $[0, 1) \cup (1, 3]$.



(a) Make a Sign Diagram for $f(x)$.

(b) Use part (a) to help you find the domain of $g(x) = \sqrt{f(x)}$

(c) Use part (a) to help you find the domain of $h(x) = [f(x)]^{-3/2}$

(d) Use part (a) to help you find the domain of $T(x) = [f(x)]^{-2/3}$

6. Recall from Section 2.1 Practice Problems:

According to the US Census, Table A-3, the share of money income in 2020 is given in the table below on the left. From these data, we can create a cumulative distribution, $y = L(x)$ called the **Lorenz Curve**.

The number $L(x)$ gives the percentage of the total national income earned by the bottom x percent of wage earners, ranked from lowest income to highest income. Since the population here is separated into 'quintiles,' each data point corresponds to 20% of the population. So, for example, $L(20)$ is the percentage of money income earned by the lowest 20% of wage earners. In this case, we see $L(20) = 3.0$. The number $L(40)$ is the percentage of the money income earned by the bottom 40% of wage earners - so this includes not only the money from the Second Quintile, but also the Lowest Quintile: $L(40) = 8.1 + L(20) = 8.1 + 3.0 = 11.1$. Likewise, $L(60)$ is the total income share of the bottom 60% of wage earners which includes the income from the Middle, Second, and Lowest Quintiles: $L(60) = 14.0 + L(40) = 14.3 + (8.1 + 3.0) = 25.1$.

Continuing in this manner, we get $L(80) = 47.7$ and $L(100) = 99.9$, which is what we would expect since 100% of the income (well rounded up) is earned by 100% of the population.

Portion of Population	Percent of Money Income
Lowest Quintile	3.0
Second Quintile	8.1
Middle Quintile	14.0
Fourth Quintile	22.6
Highest Quintile	52.2

percent wage earners, x	percent income, $L(x)$
20	3.0
40	11.1
60	25.1
80	47.7
100	99.9

(a) Use Desmos to make a table for the pairs $(x, L(x))$ and find a **power function** to fit the data:

$$y_1 \sim a\, x_1^p$$

Comment on the goodness of fit.

(b) Find and interpret $L(90)$.

(c) Use $L(x)$ to approximate the percentage of income earned by the top 5 % of the wage earners.

(d) How do your answers to parts (a) and (b) compare with those you obtained from the Section 2.1 Practice Problems?