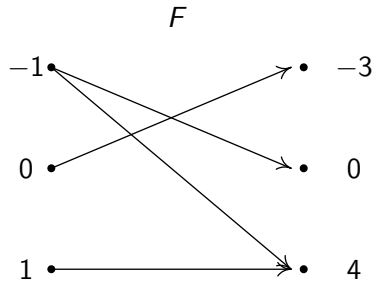
MATH 1650: TAKE HOME 01 - 20 POINTS

NAME: _____

DIRECTIONS: To receive full credit, make sure your work is neat and complete.

SECTION 1.1 PRACTICE PROBLEMS

1. Consider the mapping F below. Is F a function? Explain your reasoning.



2. Let C be the mapping that matches each student in class to their favorite color.

   Under what circumstances is C a function?

3. Let $f(x) = 2x - x^2$.

(a) Find and simplify $f(-1)$.

(b) Find and simplify an expression for $f(5x)$ and $5f(x)$.

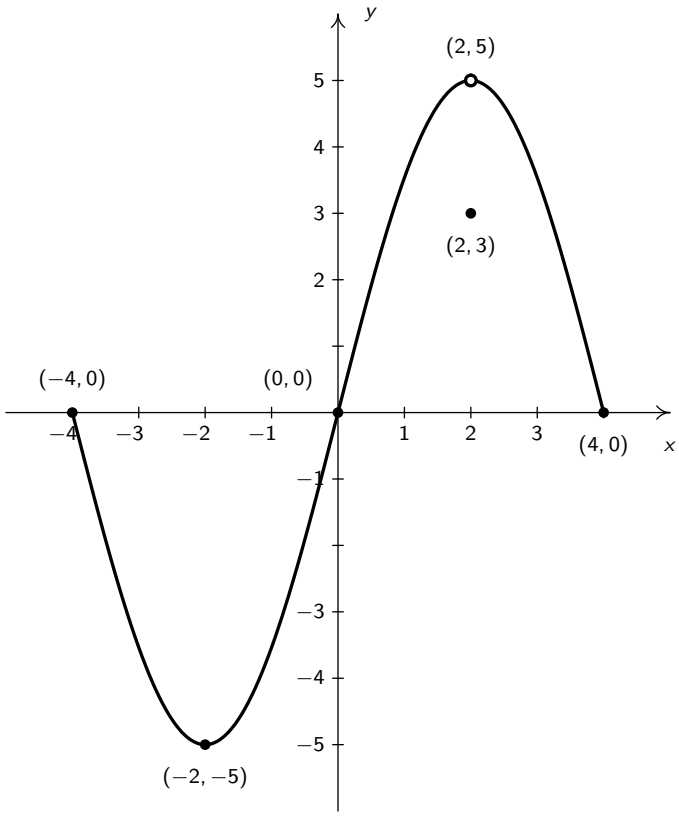
$f(5x) =$ _____ $5f(x) =$ _____

(c) Find and simplify an expression for $f(x+5)$ and $f(x)+5$.

$f(x+5) =$ _____ $f(x)+5 =$ _____

4. Answer the following questions using the graph of f below.

NOTE: Pay close attention to the 'hole' in the graph at $(2, 5)$.



The graph of $y = f(x)$.

(a) State the domain and range of f. Write your answers using interval notation.

domain: **range:**

(b) Find minimum of f.

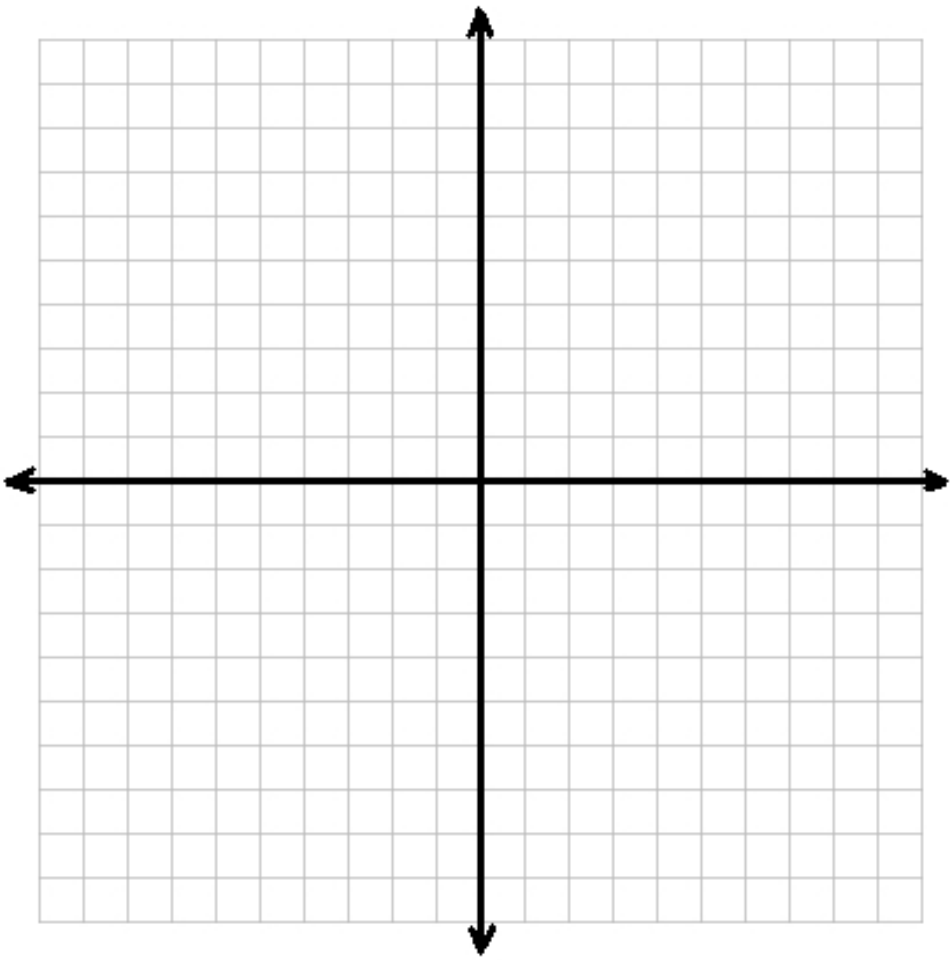
(c) Explain why f has no maximum.

(d) Find $f(2)$ and solve $f(x) = 0$.

$f(2) =$ $f(x) = 0$ when $x =$

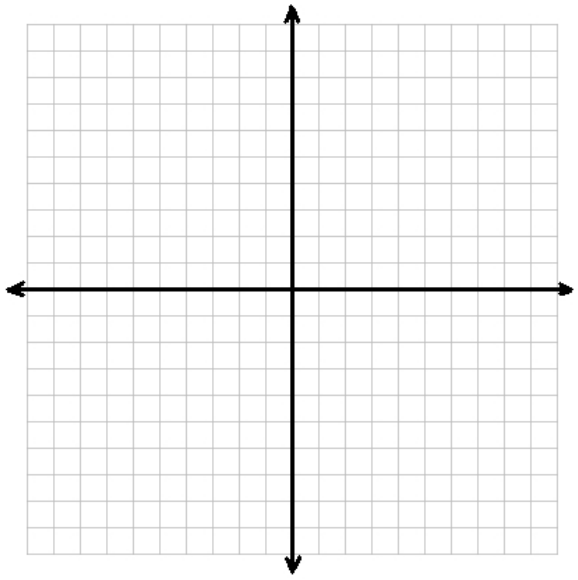
(e) Solve $f(x) < 0$. Write your answer using interval notation.

5. Sketch the graph of a function with domain $(-3, 1] \cup (2, \infty)$ and range $(-\infty, \infty)$.



SECTION 1.2 PRACTICE PROBLEMS

1. Graph $f(x) = \begin{cases} -2x - 1 & \text{if} \quad x < 1 \\ 2x - 5 & \text{if} \quad x \geq 1 \end{cases}$. Find the axis intercepts. State the domain and range.



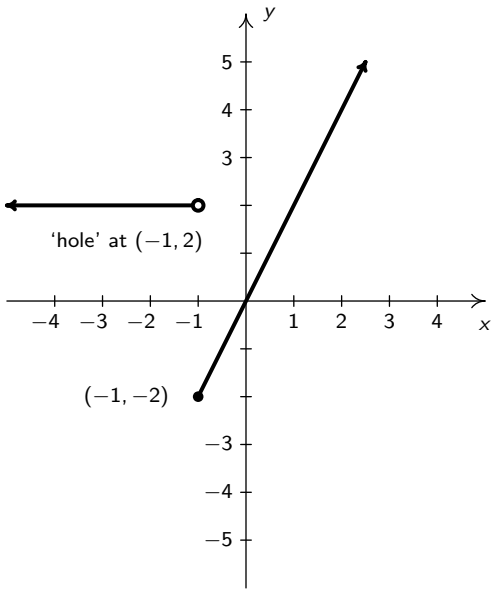
x-intercept(s):

y-intercept(s):

domain:

range:

2. Consider the graph of $y = f(x)$ below.



(a) State the interval(s), if any, over which f is:

- **increasing:**
- **decreasing:**
- **constant:**

(b) Find a possible formula for $f(x)$.

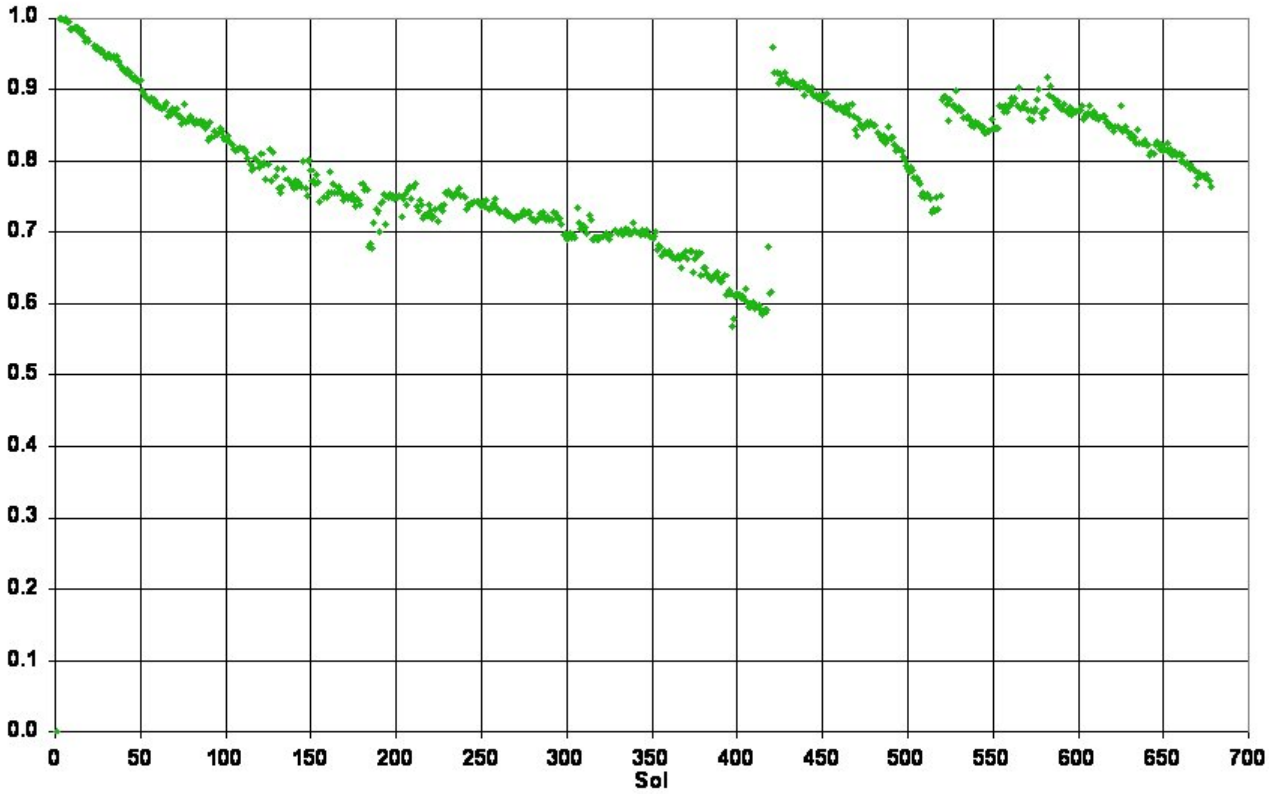
$$f(x) = \begin{cases} \_\_\_\_\_, & \text{if } \_\_\_\_\_ \\ \\ \_\_\_\_\_, & \text{if } \_\_\_\_\_ \end{cases}$$

3. Dirk's Dirt Delivery charges \$15 per cubic yard for topsoil. There is an additional \$30 delivery charge for orders under 10 cubic yards; orders of 10 cubic yards or more have free delivery. Let $D(x)$ denote the **total charge** (that is, of dirt plus delivery!) of ordering x cubic yards of topsoil from Dirk.

   Find and interpret $D(5)$ and $D(15)$ and use these to help you build a piecewise-defined formula for $D(x)$.

$$D(x) = \begin{cases} \_\_\_\_\_, & \text{if } \_\_\_\_\_ \\ \\ \_\_\_\_\_, & \text{if } \_\_\_\_\_ \end{cases}$$

4. The Mars Rover *Spirit* is powered using solar energy. As dust accumulates on the solar panels, the power output from the panels begins to decline. Below is a plot of the efficiency of the panels,[1] E as a function of the number of Martian days the rover has been active, t.



(a) Explain what the data points $(0, 1)$ and $(50, 0.9)$ mean in this situation.

Use these to find a linear function which models the efficiency, E, as a function of the number of Martian days the rover has been active, t.

NOTE: Your answer should have the form $E(t) = m\,t + b$.

[1] As measured as a fraction of the power output divided by the amount of power produced by a clean panel

(b) Use your function in number 4a to predict the efficiency after 75 days, 100 days, and 200 days. How accurate are your predictions? What could account for the error?

(c) What could account for the 'jump' in the data near Sol 425?

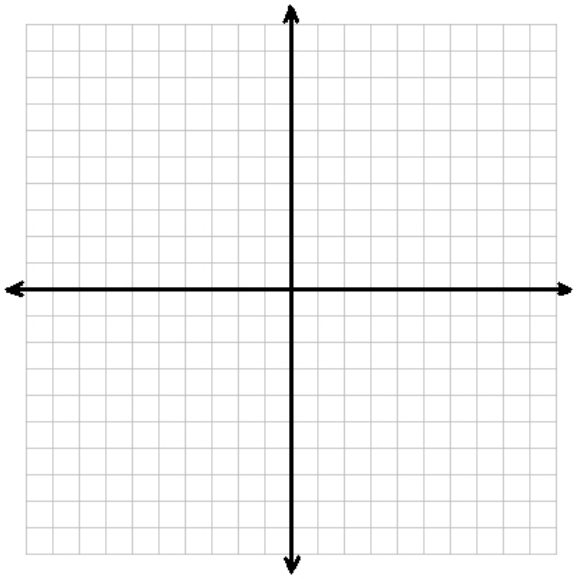
(d) Use the data points $(625, 0.85)$ and $(675, 0.75)$ to find a new linear function which models E as a function of t. According to your new model, how long will the solar panels continue to produce power?

SECTION 1.3 PRACTICE PROBLEMS

1. Rewrite each of the functions in the form $f(x) = a|x - h| + k$ to help you graph f.

   Algebraically determine the vertex, axis intercepts (if any) and state the domain, range, maximum, minimum, and interval(s) of increase / decrease / constant using interval notation.

   (a) $f(x) = |-2x + 6| + 1$



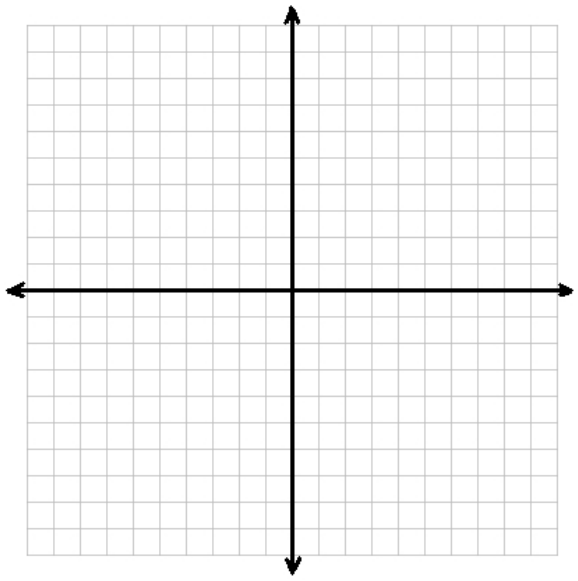
vertex: **x-intercept(s):** **y-intercept(s):**

domain: **range:** **maximum:** **minimum:**

Interval(s) over which f is:

increasing: **decreasing:** **constant:**

(b) $f(x) = 3 - \left|\frac{x-2}{4}\right|$



vertex: **x-intercept(s):** **y-intercept(s):**

domain: **range:** **maximum:** **minimum:**

Interval(s) over which f is:

increasing: **decreasing:** **constant:**

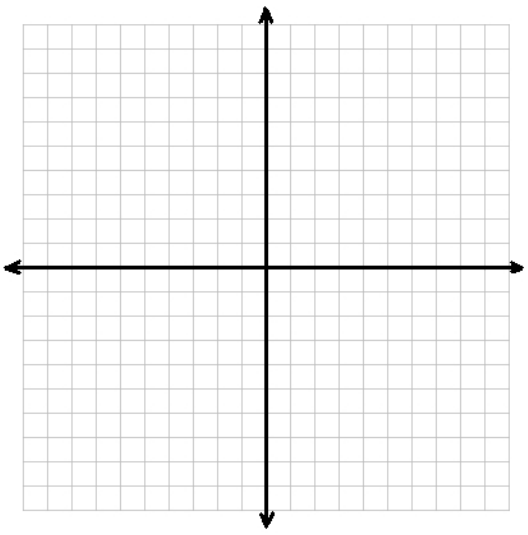
2. (a) Rewrite $y = |2x - 3|$ as a piecewise-defined function:

$$|2x-3| = \begin{cases} \_\_\_\_\_, & \text{if } \_\_\_\_\_ \\ \\ \_\_\_\_\_, & \text{if } \_\_\_\_\_ \end{cases}$$

(b) Use your answer to part (a) to help you rewrite $f(x) = 2x - |2x - 3|$ as a piecewise-defined function:

$$f(x) = \begin{cases} \_\_\_\_\_, & \text{if } \_\_\_\_\_ \\ \\ \_\_\_\_\_, & \text{if } \_\_\_\_\_ \end{cases}$$

Algebraically determine the axis intercepts of the graph of f (if any) and state the domain, range, maximum, minimum, and interval(s) of increase / decrease / constant using interval notation.



x-intercept(s): **y-intercept(s):**

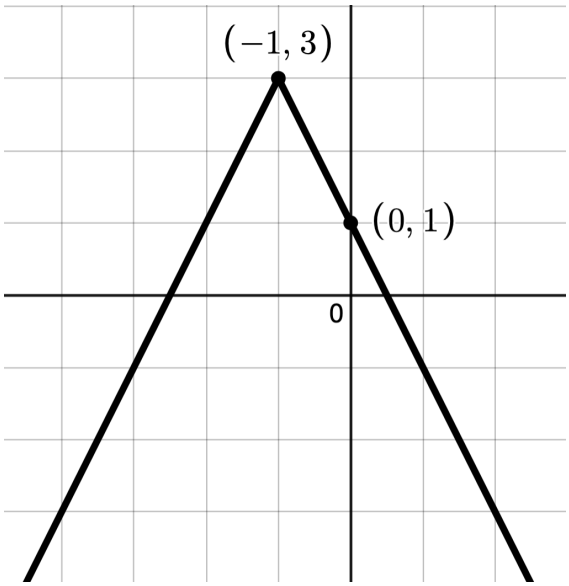
domain: **range:** **maximum:** **minimum:**

Interval(s) over which f is:

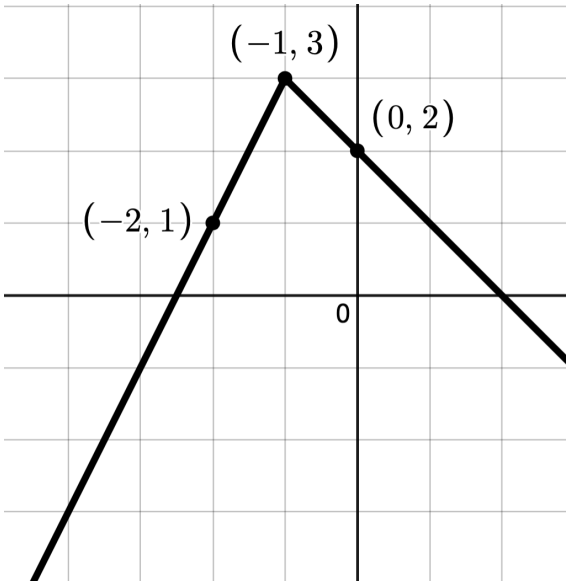
increasing: **decreasing:** **constant:**

3. Find a possible formula for the function whose graph is below of the form $f(x) = a|x - h| + k$.

   Explain your reasoning.



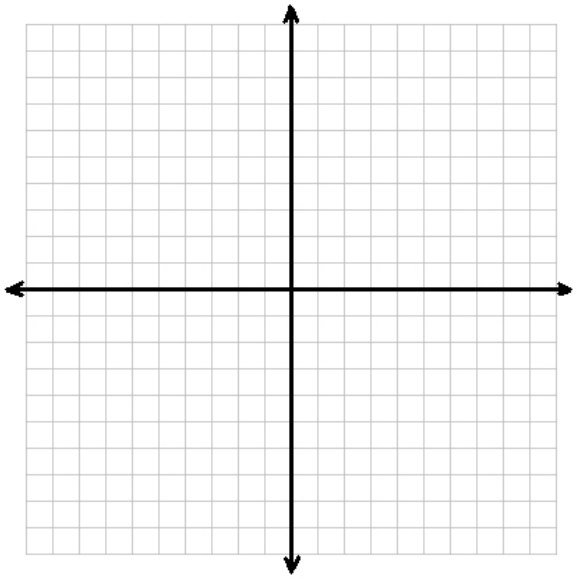
4. Explain why the function whose graph is below cannot be of the form $f(x) = a|x - h| + k$.



SECTION 1.4 PRACTICE PROBLEMS

1. Algebraically determine the vertex and axis intercepts to help you graph each function. State the domain, range, maximum, minimum, and interval(s) of increase / decrease / constant using interval notation.

   (a) $f(x) = -2(x-3)^2 + 1$



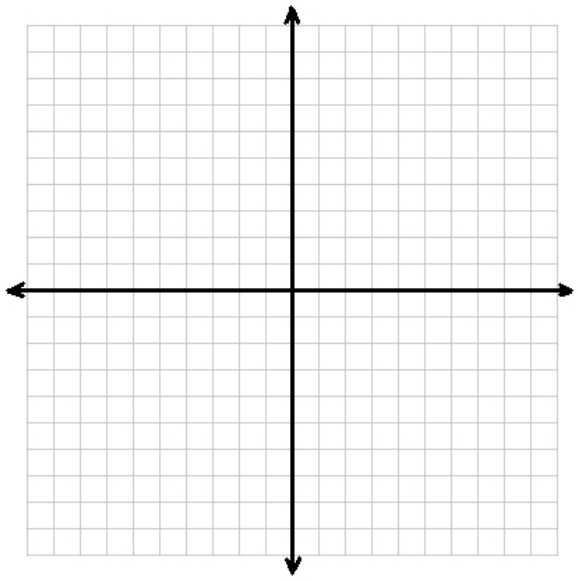
vertex: **x-intercept(s):** **y-intercept(s):**

domain: **range:** **maximum:** **minimum:**

Interval(s) over which f is:

increasing: **decreasing:** **constant:**

(b) $f(x) = x^2 - x - 6$



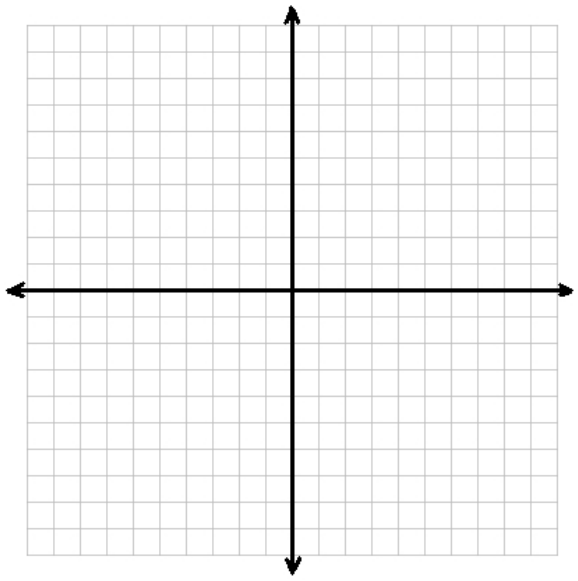
vertex: **x-intercept(s):** **y-intercept(s):**

domain: **range:** **maximum:** **minimum:**

Interval(s) over which f is:

increasing: **decreasing:** **constant:**

(c) $f(x) = x^2 - x - 5$



vertex: **x-intercept(s):** **y-intercept(s):**

domain: **range:** **maximum:** **minimum:**

Interval(s) over which f is:

increasing: **decreasing:** **constant:**

2. The temperature $T(t)$ in degrees Fahrenheit t hours after 8 AM is given by

$$T(t) = -t^2 + 10t + 40 \quad 0 \leq t \leq 12.$$

(a) Find $T(0)$ and interpret what this value means in terms of time and temperature.

(b) Find $T(5)$ and interpret what this value means in terms of time and temperature.

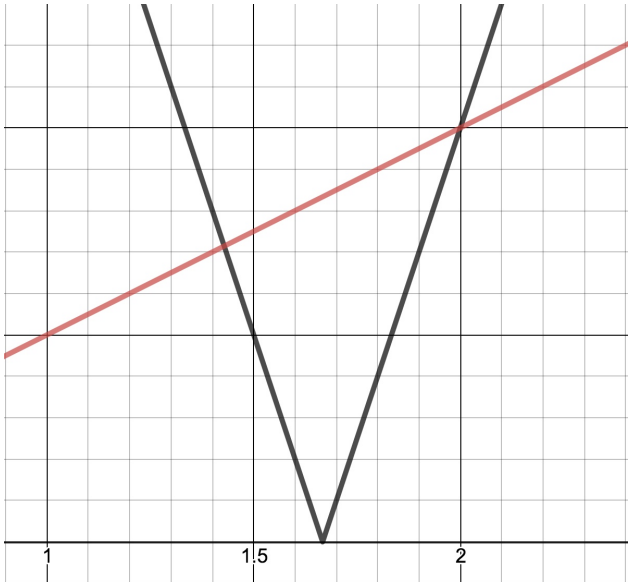
(c) Find the average rate of change of T over the interval $[0, 5]$.
Interpret what this value means in terms of time and temperature.

(d) Algebraically find the vertex of the graph $y = T(t)$.

Explain the significance of the vertex in terms of time and temperature.

SECTION 1.3 and 1.4 (INEQUALITIES) PRACTICE PROBLEMS

1. Below are graphed $f(x) = |3x - 5|$ and $g(x) = \frac{1}{2}x$ together on the axes provided.



(a) Find the intersection points of the two graphs algebraically.

(b) Use the graph to solve: $|3x - 5| \geq \frac{1}{2}x$. Write your answer using interval notation.

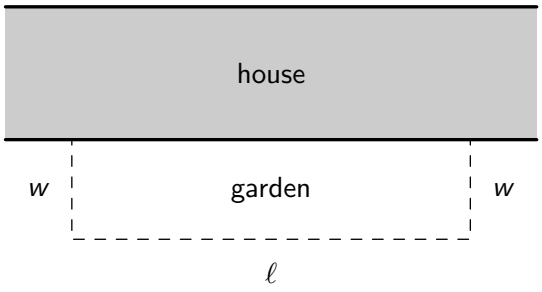
2. Use a Sign Diagram to solve: $x^2 - x \leq 12$. Write your answer using interval notation.

3. (a) Make a Sign Diagram for $f(x) = x^2 - x - 6$.

   (b) Use part (a) to help to write $g(x) = \left|x^2 - x - 6\right|$ as a three part piecewise-defined function:

$$\left|x^2 - x - 6\right| = \begin{cases} \_\_\_\_\_, & \text{if } \_\_\_\_\_ \\ \_\_\_\_\_, & \text{if } \_\_\_\_\_ \\ \_\_\_\_\_, & \text{if } \_\_\_\_\_ \end{cases}$$

4. Taylor wants to plant a garden along the side of her home. She has 200 feet of fencing to use. Since the garden will be against the house, she has no need to fence along that side of the garden.



(a) Find a formula for the area of the garden, A in terms of w and ℓ.

(b) Use the fact that Taylor has 200 feet of fencing to write an equation relating w and ℓ.

(c) Use parts (a) and (b) to get an expression for the area A as a function of only w.

What is an appropriate applied domain here?

(d) Algebraically determine the dimensions of the garden which yield the maximum area.

What is the maximum area?

Dimensions of the garden: $w =$ _____, $\ell =$ _____, maximum area: _____