MATH 2700: TEST 04 (100 points.)

NAME: ______________________________

DIRECTIONS: Show all work.

1. Consider the iterated integral: $\displaystyle\int_0^2 \int_{\frac{y}{2}}^1 \cos\left(x^2\right)\, dx\, dy$

   (a) Sketch the region over which this integral is being evaluated.

   (b) Rewrite this integral with order $dy\, dx$.

   (c) Evaluate the integral in part (b).

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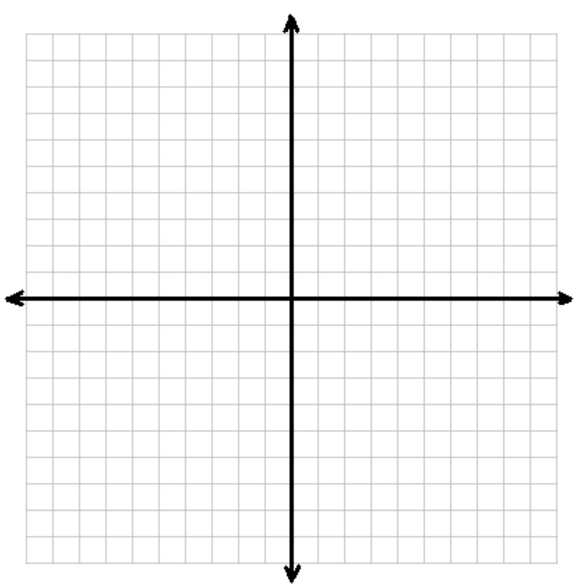
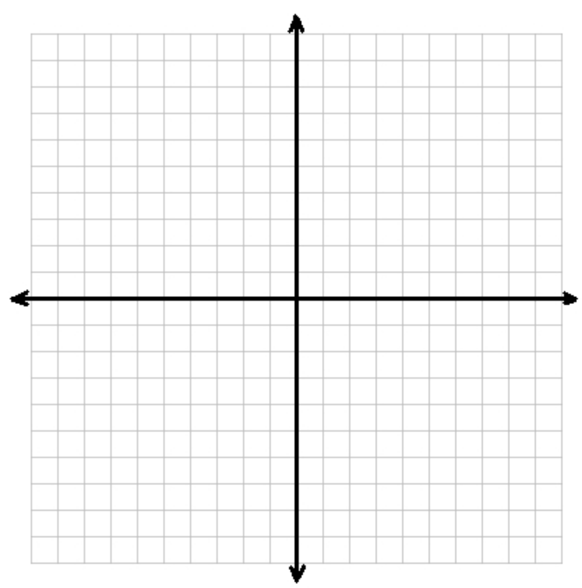
2. Let Q be the solid bounded by the cone $z = 2 - \sqrt{x^2 + y^2}$ and the coordinate planes in the first octant.

(a) Sketch the projection of Q in each of the the xy-plane and the yz-planes.

Be sure to label all the bounding curves.



projection into xy-plane



projection into yz-plane

Set-up iterated integrals with the given requirements that calculate the **volume** of Q .

DO NOT EVALUATE THE INTEGRALS

(b) a **double** iterated integral with order $dy\,dx$.

(c) a **triple** iterated integral with order $dz\,dy\,dx$.

(d) a **double** iterated integral in **polar coordinates**.

(e) a **triple** iterated integral in **cylindrical coordinates**.

(f) a **double** iterated integral with order $dz\,dy$.

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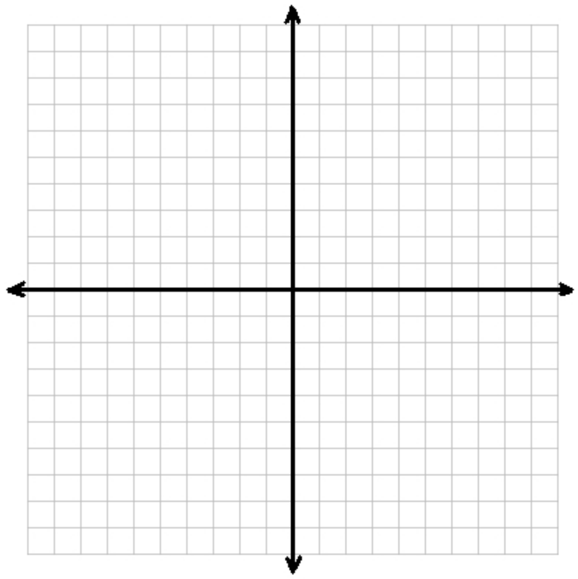
3. Consider the sum of iterated integrals in rectangular coordinates: $\displaystyle\int_0^3 \int_0^x x\, dy\, dx + \int_3^6 \int_0^{\sqrt{6x-x^2}} x\, dy\, dx.$

(a) Sketch the region over which these integrals are being evaluated in the xy-plane below.



(b) Convert the sum of integrals to a single integral in **polar coordinates**.

DO NOT EVALUATE THE INTEGRAL

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4. Let Q be the 'sno$^{\text{TM}}$ cone' - the solid below $z = \sqrt{8 - x^2 - y^2}$ and above $z = \sqrt{x^2 + y^2}$.

   Set-up **but do not evaluate** triple iterated integral integrals that calculate the volume of Q:

   (a) using **rectangular** coordinates.

   (b) using **cylindrical** coordinates.

   (c) using **spherical** coordinates.

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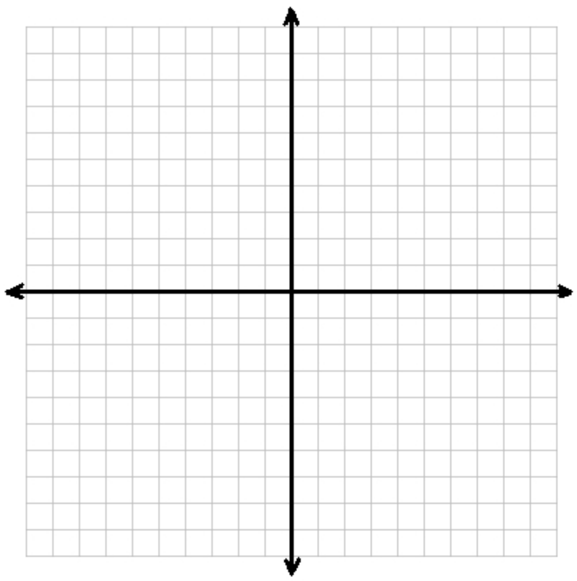
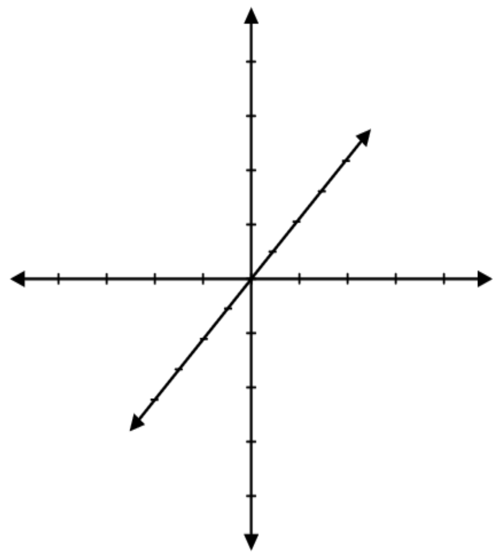
5. Consider the triple integral:

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{25-x^2-y^2}} z \, dz \, dy \, dx$$

(a) sketch or otherwise describe the solid over which this integral is being evaluated, Q.





(b) Convert the integral to **cylindrical coordinates** and **evaluate** the integral.

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(c) Convert the integral to **spherical coordinates** but **DO NOT** evaluate the integral.

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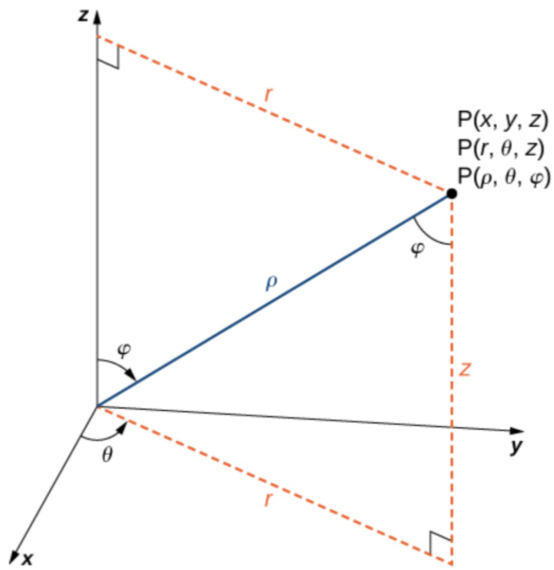
(d) Interpret the integrals below as moments of the solid Q to evaluate them. Explain your reasoning.

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{25-x^2-y^2}} x\,dz\,dy\,dx \quad \text{and} \quad \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{25-x^2-y^2}} y\,dz\,dy\,dx$$

(e) **BONUS:** Find the center of mass Q assuming a uniform density given that:

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{25-x^2-y^2}} dz\,dy\,dx = \frac{122\pi}{3}$$

FORMULA PAGE



CONVERSION BETWEEN RECTANGULAR AND CYLINDRICAL COORDINATES:

$x = r\cos(\theta)$ $\quad$ $y = r\sin(\theta)$ $\quad$ $z = z$

$r^2 = x^2 + y^2$ $\quad$ $\tan(\theta) = \frac{y}{x},\ x \neq 0$ $\quad$ $z = z$

CONVERSION BETWEEN RECTANGULAR AND SPHERICAL COORDINATES:

$x = \rho\sin(\phi)\cos(\theta)$ $\quad$ $y = \rho\sin(\phi)\sin(\theta)$ $\quad$ $z = \rho\cos(\phi)$

$\rho^2 = x^2 + y^2 + z^2$ $\quad$ $\tan(\theta) = \frac{y}{x},\ x \neq 0$ $\quad$ $\cos(\phi) = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$

CONVERSION BETWEEN CYLINDRICAL AND SPHERICAL COORDINATES:

$r = \rho\sin(\phi)$ $\quad$ if $r \geq 0,\ \theta = \theta$ $\quad$ $z = \rho\cos(\phi)$

$\rho^2 = r^2 + z^2$ $\quad$ if $r \geq 0,\ \theta = \theta$ $\quad$ $\cos(\phi) = \frac{z}{\sqrt{r^2 + z^2}}$