MATH 2700: TAKE HOME 04 (50 points.)

NAME: ______________________________

DUE: The day of Test 4, at the beginning of class.

DIRECTIONS: Show all work.

1. Consider the iterated integral: $\int_0^1 \int_1^2 y \cos(\pi\, xy)\, dy\, dx$

   (a) Sketch the region over which this integral is being evaluated.

   (b) Rewrite this integral with order $dx\, dy$.

   (c) Evaluate the integral in part (b).

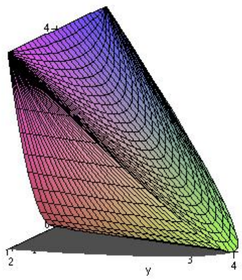
2. Consider the iterated integral: $\displaystyle\int_0^2 \int_{\frac{x}{2}}^1 e^{-y^2}\, dy\, dx.$

(a) Sketch the region over which this integral is being evaluated.

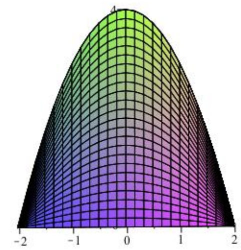
(b) Rewrite this integral with order $dx\, dy$.

(c) Evaluate the integral in part (b).

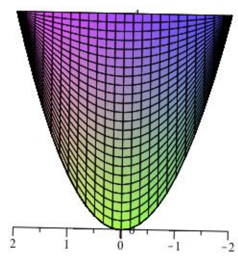
3. Let Q be the solid bounded by the parabolic cylinder $z = x^2$, the plane $y + z = 4$, and the $xz-$plane. Set-up, **<u>but do not evaluate</u>**, double iterated integrals in the prescribed order which would find the volume of Q. Below are some screenshots which may be of help...



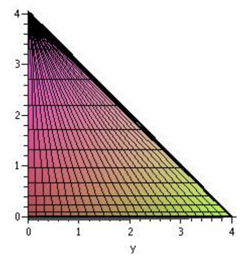
A 'standard' view



A view down the positive z-axis.



A view down the positive y-axis



A view down the positive x-axis.

(a) Integration order: $dy\,dx$

(b) Integration order: $dy\,dz$

(c) Integration order: $dx\,dz$

4. Follow the steps below to show $\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\,dx\,dy \neq \int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\,dy\,dx$

(a) Integrate: $\int_0^1 \frac{x-y}{(x+y)^3}\,dx$ using the substitution $u = x + y$ (or, equivalently, $x = u - y$.)

NOTE: This means: $du = dx$, when $x = 0$, $u = y$, when $x = 1$, $u = y + 1$.

(b) Show $\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\,dx\,dy = -\frac{1}{2}$ using your answer to part (a)

(c) Explain why $\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dx\,dy = \int_0^1\int_0^1 \frac{u-v}{(u+v)^3}\,du\,dv = \int_0^1\int_0^1 \frac{y-x}{(x+y)^3}\,dy\,dx.$

(d) Use your answer to 3b and your observation above to show:

$$\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dy\,dx = -\int_0^1\int_0^1 \frac{y-x}{(x+y)^3}\,dy\,dx = -\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dx\,dy = +\frac{1}{2}$$

(e) You've just shown that $\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dx\,dy \neq \int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dy\,dx$!

What doesn't Fubini's Theorem apply?

5. Convert the integral given from rectangular to polar coordinates to evaluate:

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} y \, dy \, dx$$

6. Let R be the region on the xy-plane above $y = x$ but below $y = \sqrt{8x - x^2}$.

   Set-up, <u>but do not evaluate</u>, a double iterated integral in polar coordinates which would find the area of R.

7. Let Q be the solid bounded by the surfaces $z = x^2 + y^2$ and $z = 6x$.

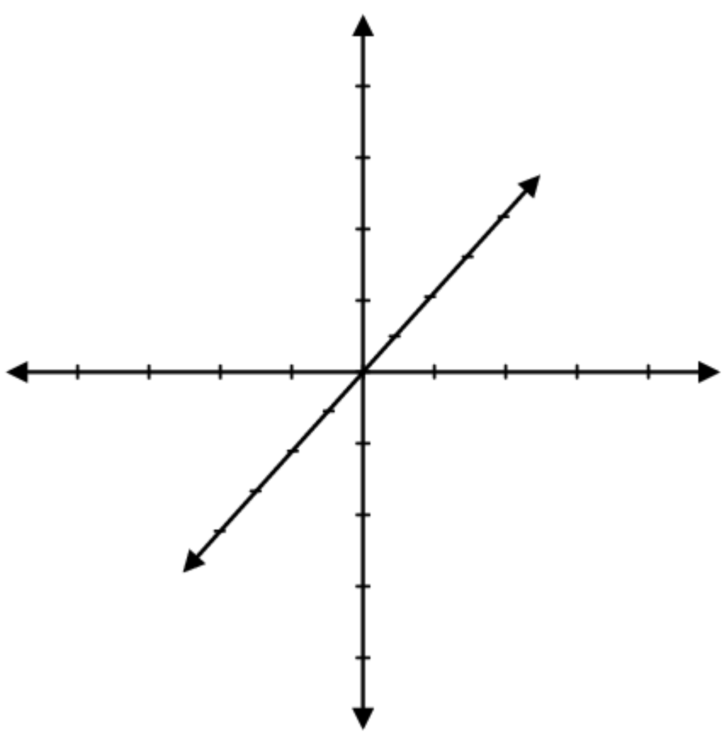
(a) Set-up a double iterated integral in **polar coordinates** that would find the **volume** of Q.

(b) Evaluate the integral from part (a) to find the volume of Q.

HINT: If n is even: $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^n(\theta)\, d\theta = \pi \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\cdots\left(\frac{n-1}{n}\right)$

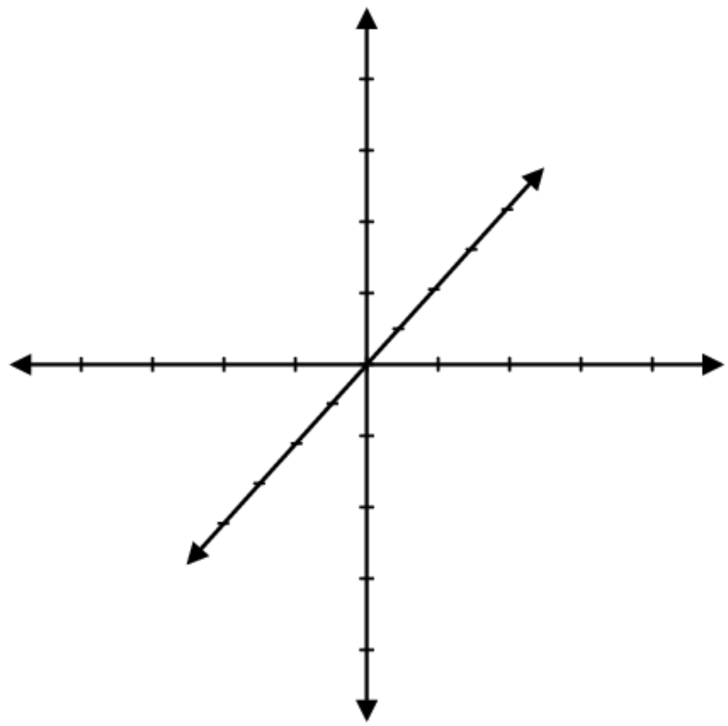
8. Sketch or otherwise describe the solid whose volume is found by $\int_0^3 \int_0^{2-\frac{2}{3}x} \int_0^{6-2x-3y} dz\, dy\, dx$

Explain your reasoning.



9. Consider the integral: $N = \int_{-2}^{2} \int_{y^2}^{4} \int_{x}^{4} xz \, dz \, dx \, dy$

(a) Sketch or otherwise describe the solid over which N is being evaluated. Explain your reasoning.



(b) Rewrite the integral in the order: $dy \, dx \, dz$.

10. Let Q be the solid bounded by $z = \sqrt{4 - x^2 - y^2}$, $z = \sqrt{1 - x^2 - y^2}$, and the xy-plane with the space inside $z = \sqrt{3x^2 + 3y^2}$ removed. Find the **volume** of Q by evaluating an integral in **spherical** coordinates.

HINT: A good cross-section sketch goes a long way!

11. Sketch the solid whose volume is found by the sum of the following integrals in spherical coordinates:

$$\int_0^{2\pi}\int_0^{\pi/6}\int_0^{3\sec(\phi)} \rho^2 \sin(\phi)\ d\rho\, d\phi\, d\theta + \int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{\csc(\phi)\cot(\phi)} \rho^2 \sin(\phi)\ d\rho\, d\phi\, d\theta$$

Explain your reasoning.

12. Convert the integral below from rectangular to (a) cylindrical and (b) spherical coordinates:

$$\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \int_{3\sqrt{x^2+y^2}}^{6} z \, dz \, dy \, dx$$

DO NOT EVALUATE THE INTEGRALS

(a) cylindrical coordinates.

(b) spherical coordinates.

13. Let Q be the solid bounded by $z = 4 - x^2 - y^2$ and the xy-plane which lies in the **first octant**.

If the density of Q is $\rho(x, y) = xy$, find the **mass** of Q by evaluating an integral in **cylindrical coordinates**.