MATH 2700: TEST 03 (100 points.)

NAME: ______________________________

DIRECTIONS: Show all work.

1. Consider $z = f(x, y) = \sin\left(x^2 + y^2\right)$.

   (a) Find two level curves: $z = f(x, y) = \frac{1}{2}$. What are they?

   (b) Explain why there are **infinitely** many level curves: $z = f(x, y) = \frac{1}{2}$.

   (c) Explain why there are **no** level curves: $z = f(x, y) = 2$.

   (d) In general, for which values of c are there level curves $z = f(x, y) = c$? Explain.

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2. Let $f(x, y) = \dfrac{3xy}{x^2 + y^2}$.

(a) Find $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$ along the path $y = a\,x^2$.

(b) Based on your answer to (a), what, if anything, can you conclude about $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$?

(c) Convert to polar coordinates: $x = r\cos(\theta)$ and $y = r\sin(\theta)$ to help you find $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$.

HINT: Remember: $(x, y) \to (0, 0)$ is equivalent to $r \to 0$.

(d) Explain how your answer to part (c) proves $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist.

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3. Consider the limit: $\lim\limits_{(x,y)\to(0,0)} 4\,|xy|\,\cos\left(\dfrac{\pi xy}{3x^2+y}\right)$

(a) Explain why we cannot use direct substitution to evaluate this limit.

(b) Use the Squeeze Theorem to evaluate this limit.

HINT: Remember: $-1 \leq \cos(\theta) \leq 1$ for all angles θ.

4. Let $f(x, y) = \sqrt{x^2 + y^2}$.

(a) Find and simplify $f_x(x, y)$ and $f_y(x, y)$ and explain how these show f is differentiable at $(3, -4)$.

(b) Find $\nabla f(3, -4)$ and show $\nabla f(3, -4)$ is, in this case, a **unit** vector.

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(c) What is the maximum value of $D_{\hat{u}}f(3,-4)$? What is $\hat{u}$ in this case?

(d) What is the minimum value of $D_{\hat{u}}f(3,-4)$? What is $\hat{u}$ in this case?

(e) Find the equation of the tangent plane to the surface $z = f(x, y)$ at $(x, y) = (3, -4)$.

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5. Consider the surface: $\left(2-\sqrt{x^2+y^2}\right)^2+z^2=2.$

Find the equation of the plane tangent to the surface at the point $(-1,0,1)$.

6. The volume of a right cylinder, V is related to the radius of the base, r and the height, h by the formula:

$$V = \pi r^2 h$$

(a) Calculate the volume of a cylinder V using the measurements $r = 2$ mm and $h = 5$ mm.

Leave your answer in terms of π.

(b) Find an expression for the differential dV in terms of r, h, dr, and dh.

(c) If the measurements for r and h were accurate to within ± 0.01 mm, approximate the percent relative propagated error, $\frac{dV}{V}$, in calculating the volume of the cylinder using $r = 2$ mm and $h = 5$ mm.

7. Show $f(x,y) = e^{-x}\cos(y)$ is a solution to the Laplace Equation:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

8. Suppose $z = f(x,y)$ where $x = r\cos(\theta)$ and $y = r\sin(\theta)$.

   (a) Draw a tree of dependence connecting z with r and θ.

   (b) Use the Chain Rule to write an expression for $\frac{\partial z}{\partial r}$ involving $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, r, and/or θ.

9. Find and classify all the local maximums, minimums, and saddle points of: $z = 8y^3 - x^3 - 6xy$.

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10. Use the Lagrange Multipliers to minimize $F(x, y, z) = x^2 + y^2 + z^2$ subject to $10x + y + 7z = 29$.

NOTE: You solved this problem a different way on Take Home 01 ...