MATH 2600: TEST 01 (100 POINTS)

NAME: ____

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

1. Find the following integrals:

   (a) $\displaystyle\int \frac{(\ln(x))^2}{x}\,dx$

   (b) $\displaystyle\int \frac{\ln(x)}{x^2}\,dx$

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2. Find the following integrals:

(a) $\int \cos^4(\theta)\, d\theta$

(b) $\int \tan^4(\theta)\, d\theta$

3. Find the following integrals:

(a) $\displaystyle\int \frac{x^2 - 2x + 4}{x^2 + 4}\, dx$

(b) $\displaystyle\int \frac{x^2 - 2x + 4}{x^3 + 4x}\, dx$

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4. Consider the integral: $\int x\sqrt{9-x^2}\,dx$

(a) Evaluate this integral using the substitution: $u = 9 - x^2$.

(b) Evaluate this integral using a trigonometric substitution.

5. Consider the integral: $\int x\sqrt{9-x}\,dx$

(a) Evaluate this integral using the substitution: $u = 9 - x$.

(b) Evaluate this integral using parts with: $u = x$ and $dv = \sqrt{9-x}\,dx$.

BONUS: on the back of this page, algebraically show your answers to (a) and (b) are the same.

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BASIC INTEGRATION FORMULAS:

- $\int du = \int 1\, du = u + C$
- $\int u^p\, du = \frac{1}{p+1} u^{p+1} + C, \quad p \neq -1$
- $\int \sin(u)\, du = -\cos(u) + C$
- $\int \cos(u)\, du = \sin(u) + C$
- $\int \csc(u)\cot(u)\, du = -\csc(u) + C$
- $\int \sec(u)\tan(u)\, du = \sec(u) + C$
- $\int \csc^2(u)\, du = -\cot(u) + C$
- $\int \sec^2(u)\, du = \tan(u) + C$
- $\int \frac{1}{u}\, du = \ln|u| + C$
- $\int e^u\, du = e^u + C$

RECALL: If $a > 0$:

- $\int \frac{1}{\sqrt{a^2 - u^2}}\, du = \arcsin\left(\frac{u}{a}\right) + C = \sin^{-1}\left(\frac{u}{a}\right) + C$
- $\int \frac{1}{u^2 + a^2}\, du = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$
- $\int \frac{1}{u\sqrt{u^2 - a^2}}\, du = \frac{1}{a}\text{arcsec}\left(\frac{|u|}{a}\right) + C = \frac{1}{a}\sec^{-1}\left(\frac{|u|}{a}\right) + C$

BASIC INTEGRATION PROPERTIES:

Suppose F and G are antiderivatives of f and g, respectively, on some open interval I.

- **CONSTANT MULTIPLE RULE:** $\int k \cdot f(x)\, dx = k \int f(x)\, dx = k \cdot F(x) + C$
- **SUM AND DIFFERENCE RULE:** $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx = F(x) \pm G(x) + C$
- **LINEAR SUBSTITUTION:** If $a \neq 0$, then $\int f(ax + b)\, dx = \frac{1}{a} F(ax + b) + C$.

BASIC IDENTITIES:

- To convert sines into cosines: $\sin^2(t) = 1 - \cos^2(t)$; moreover, $\sin^{2k}(t) = \left(\sin^2(t)\right)^k = \left(1 - \cos^2(t)\right)^k$

- To convert cosines into sines: $\cos^2(t) = 1 - \sin^2(t)$; moreover, $\cos^{2k}(t) = \left(\cos^2(t)\right)^k = \left(1 - \sin^2(t)\right)^k$

- The so-called 'power reduction' formulas (a.k.a. 'double angle' formulas):

$$\sin^2(t) = \frac{1 - \cos(2t)}{2}; \quad \text{moreover,} \quad \sin^{2k}(t) = \left(\sin^2(t)\right)^k = \left(\frac{1 - \cos(2t)}{2}\right)^k$$

$$\cos^2(t) = \frac{1 + \cos(2t)}{2}; \quad \text{moreover,} \quad \cos^{2k}(t) = \left(\cos^2(t)\right)^k = \left(\frac{1 + \cos(2t)}{2}\right)^k$$

- To convert secants into tangents: $\sec^2(t) = 1 + \tan^2(t)$; moreover, $\sec^{2k}(t) = \left(\sec^2(t)\right)^k = \left(1 + \tan^2(t)\right)^k$

- To convert tangents into secants: $\tan^2(t) = \sec^2(t) - 1$; moreover, $\tan^{2k}(t) = \left(\tan^2(t)\right)^k = \left(\sec^2(t) - 1\right)^k$