MATH 2600: TAKE HOME 01 (20 POINTS)

NAME: ______________________________

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

SECTION 8.2 PRACTICE PROBLEMS

1. Integration by parts can be thought of reversing which derivative rule?

2. (a) What does the 'L' stand for in the acronym: 'L.I.A.T.E.'?

   (b) Find the integral: $\int \sqrt{x}\,\ln(x)\,dx$

3. (a) What does the 'I' stand for in the acronym: 'L.I.A.T.E.'?

   (b) Find the integral: $\int \sin^{-1}(x)\,dx$

4. Find the following integrals.

(a) $\displaystyle\int 2t \cos\left(t^2\right)\, dt$

(b) $\displaystyle\int t^2 \cos(2t)\, dt$

5. (a) Use integration by parts to show: $\displaystyle\int x^p \ln(x)\, dx = \frac{x^{p+1}\ln(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2} + C$

For which values of p is your formula valid? Explain.

(b) Find $\displaystyle\int x^{-1} \ln(x)\, dx$.

6. Assuming s and ω are positive constants, use integration by parts (twice!) to show:

$$\int e^{-st} \cos(\omega t)\, dt = \frac{e^{-st}\left[\omega \sin(\omega t) - s \cos(\omega t)\right]}{s^2 + \omega^2} + C$$

Start with $u = \cos(\omega t)$ and $dv = e^{-st}\, dt$ ...

SECTION 8.3 PRACTICE PROBLEMS

1. Convert the given expression from one involving $\sin(3x)$ to one involving $\cos(3x)$:

   $\sin^8(3x) =$

2. If we want to make a substitution $u = \sec(\theta)$, we 'keep back' a factor of $du =$ ____________________ $d\theta$

3. Find: $\displaystyle\int \cos^3(5x)\,\sqrt{\sin(5x)}\,dx$

4. Find: $\displaystyle\int_0^{\frac{\pi}{4}} \sec^4(\theta)\,d\theta$

5. Derive the following 'reduction' formula by working through the parts below:

$$\int \tan^n(x)\,dx = \frac{1}{n-1}\tan^{n-1}(x) - \int \tan^{n-2}(x)\,dx, \quad \text{for } n > 2.$$

(a) Write $\tan^n(x) = \tan^{n-2}(x)\,\tan^2(x)$ and fill in the steps to show:

$$\int \tan^n(x)\,dx = \ldots = \int \tan^{n-2}(x)\,\sec^2(x)\,dx - \int \tan^{n-2}(x)\,dx$$

(b) Integrate: $\int \tan^{n-2}(x)\,\sec^2(x)\,dx$ to complete the formula!

SECTION 8.4 PRACTICE PROBLEMS

1. Consider the expression $\sqrt{x^2 - 9}$.

   - True or False: $\sqrt{x^2 - 9} = x - 3$.

   - True or False: $\sqrt{x^2 - 9} = |x| - 3$.

   - Let $x = 3\sec(\theta)$. Show that $\sqrt{x^2 - 9}$ simplifies to $3|\tan(\theta)|$.

2. Use a trigonometric substitution as demonstrated in class to find: $\displaystyle\int_0^3 (x^2 + 9)^{-3/2}\, dx$

3. Consider the integral: $\int (x^2 - 9)^{-3/2}\, dx$.

(a) Evaluate this integral using a trigonometric substitution as demonstrated in class.

You may assume $x > 3$.

(b) Does your answer change if $x < -3$? If so, how? If not, why not?

4. Consider the integral: $\displaystyle\int_{-4}^{-1} \sqrt{8 - 2x - x^2}\, dx$.

(a) Find the value of this integral using a trigonometric substitution as demonstrated in class.

HINT: Complete the square and don't forget to change the limits on the integral from x's to θ's!

(b) Interpret $\displaystyle\int_{-4}^{-1} \sqrt{8 - 2x - x^2}\, dx$ graphically and check your answer using a formula from geometry.

SECTION 8.5 PRACTICE PROBLEMS

1. Write the **form** of the partial fraction decomposition below. Do **not** find the values of the coefficients!

$$\frac{2x+7}{5x^3(x^2+4)} =$$

2. Find: $\displaystyle\int \frac{3x^2 - x + 3}{x-1}\,dx$

3. (a) Find: $\int \frac{2x-1}{x^2+9}\,dx$

(b) Find: $\int \frac{2x-1}{x^2+9x}\,dx$

SECTION 8.7 PRACTICE PROBLEMS

1. Use one of the identities below to help you find: $\displaystyle\int_0^{\frac{\pi}{2}} \sin(x)\,\cos(3x)\,dx$.

   - $\sin(A)\,\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$
   - $\cos(A)\,\cos(B) = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$
   - $\sin(A)\,\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

2. Use the substitution $z = \tan\left(\frac{x}{2}\right)$ to help you evaluate: $\displaystyle\int_0^{2\pi/3} \frac{1}{\cos(x)+\sin(x)+1}\,dx$.

   Some helpful formulas: if $z = \tan\left(\frac{x}{2}\right)$, then:

   $dx = \dfrac{2}{1+z^2}\,dz$ $\qquad\cos(x) = \dfrac{1-z^2}{1+z^2}$ $\qquad\sin(x) = \dfrac{2z}{1+z^2}$

   HINT: Don't forget to change the limits of the integral from x's to z's!

3. Find the integral $\displaystyle\int \frac{1}{x\,(x-1)^{\frac{3}{2}}}\,dx$ using the substitution $u = (x-1)^{\frac{1}{2}}$.

HINT: If $u = (x-1)^{\frac{1}{2}}$, then $(x-1)^{\frac{3}{2}} = \left((x-1)^{\frac{1}{2}}\right)^3 = u^3$, and $x = u^2 + 1$ so that $dx = 2u\,du$ ...

4. **EXPLORATION:** Consider the integral: $\displaystyle\int \frac{1}{x^2 - a^2}\, dx$ where $a > 0$ is a constant.

(a) Evaluate this integral using the method of partial fractions.

What does WolframAlpha tell you this integral equals?

(b) Recall the hyperbolic functions $\tanh(t)$ and $\operatorname{sech}(t)$ satisfy the following properties:

$$\tanh^2(t) + \operatorname{sech}^2(t) = 1, \quad D_t[\tanh(t)] = \operatorname{sech}^2(t)$$

Also recall that $f(t) = \tanh(t)$ has domain and range $(-\infty, \infty)$ and is invertible: $f^{-1}(t) = \tanh^{-1}(t)$.

Let $x = a\tanh(t)$.

i. Show that $x^2 - a^2 = -a^2\operatorname{sech}^2(t)$.

ii. Calculate dx.

iii. Find $\displaystyle\int \frac{1}{x^2 - a^2}\, dx$ using the substitution $x = a\tanh(t)$. Write your final answer in terms of x.