MATH 2600: TEST 04 (100 POINTS)

NAME: _____

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

1. Let R be the region in the first quadrant bounded by $y = 4 - x$ and $y = \dfrac{3}{x}$.

   (a) Sketch a detailed graph of R. Make sure you label the intersection points.

   (b) Find the area of R.

(c) Set-up integral(s) which would compute the volume of the solid obtained by revolving R about:

DO NOT EVALUATE THE INTEGRALS

i. x-axis

ii. y-axis

iii. the line $x = 4$.

2. Use a definite integral to find the arc length of the graph of $f(x) = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{1/2}$ over the interval $[1, 4]$.

   HINT: Yes, the algebra does work out ...

3. Consider the polar curve: $r = 1 - \sin(\theta)$.

(a) Find and simplify an expression for the arc length differential, ds.

(b) Write an integral which would compute the arc length of this polar curve.

DO NOT EVALUATE THE INTEGRAL

4. Let C be the curve traced out by the parametric equations:

$$\begin{cases} x &= 2\cos(t) + 1 \\ y &= 2\sin(t) - 3 \end{cases}, \quad 0 \leq t \leq 2\pi$$

(a) Find and simplify the arc length differential, ds.

(b) Find the arc length of C using you answer to part (a).

(c) **BONUS:** Prove that C is a circle and check your answer to part (b) using a formula from geometry.

5. Suppose the work done stretching a spring 1.5 meters from its equilibrium position is 9 J.

(a) Find the spring constant, k. Be sure to include units.

(b) Find the work done stretching the spring **an additional** 1.5 meters.

(c) Why is your answer to (b) greater than 9 J? Shouldn't the answer be the same as what we had when we stretched the spring the original 1.5 m?

6. An above ground circular pool has a diameter of 24 feet with a depth of 4 feet. Assuming the water in the pool is 1 foot deep, use an integral calculate the work done pumping the water to the top of the pool. Develop the integral as we did in class. (Assume the weight-density of water is 62.4 pounds per cubic foot.)

BONUS: Check your answer by doing a center of mass calculation