MATH 2600: TEST 03 (100 POINTS)

NAME: _____

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

1. (a) Find the quadratic approximation to $f(x) = \ln(x)$ centered at $x = 1$.

(b) Use part (a) to help you approximate $\ln(2)$.

(c) Use Taylor's Remainder Theorem to help you approximate the error in your approximation in part (b).

2. Find the interval of convergence for the power series: $f(x) = \sum_{k=1}^{\infty} \frac{(-1)^k (x-3)^k}{k\, 2^{k+1}}$.

3. (a) Use the Maclaurin Series for $f(x) = \cos(x)$ to find the Maclaurin Series for $g(x) = \cos\left(x^2\right)$.

(b) Use your answer to part (a) to find a series representation of $\int_0^1 \cos\left(x^2\right)\, dx$.

(c) Use the AST Remainder Theorem to approximate the value of $\int_0^1 \cos\left(x^2\right)\, dx$ to three decimal places.

4. The quantity $L(x) = \dfrac{1}{\sqrt{1-x^2}}$ is called the **Lorentz Factor** and is important in relativistic calculations.

   Here x represents the ratio of an object's speed to the speed of light.

   (a) Use the Binomial Series to find a **quadratic approximation** of $L(x)$ centered at 0.

   HINT: $L(x) = \dfrac{1}{\sqrt{1-x^2}} = \left(1-x^2\right)^{-1/2}$

   (b) Approximate $L(0.5)$ and $L(0.75)$ using your answer to part (a).

5. Let C be the curve traced out by: $\{x = \sin(2t),\ y = \sin(3t),\ 0 \leq t \leq 2\pi$.

(a) Find all values of t at which C passes through the origin.

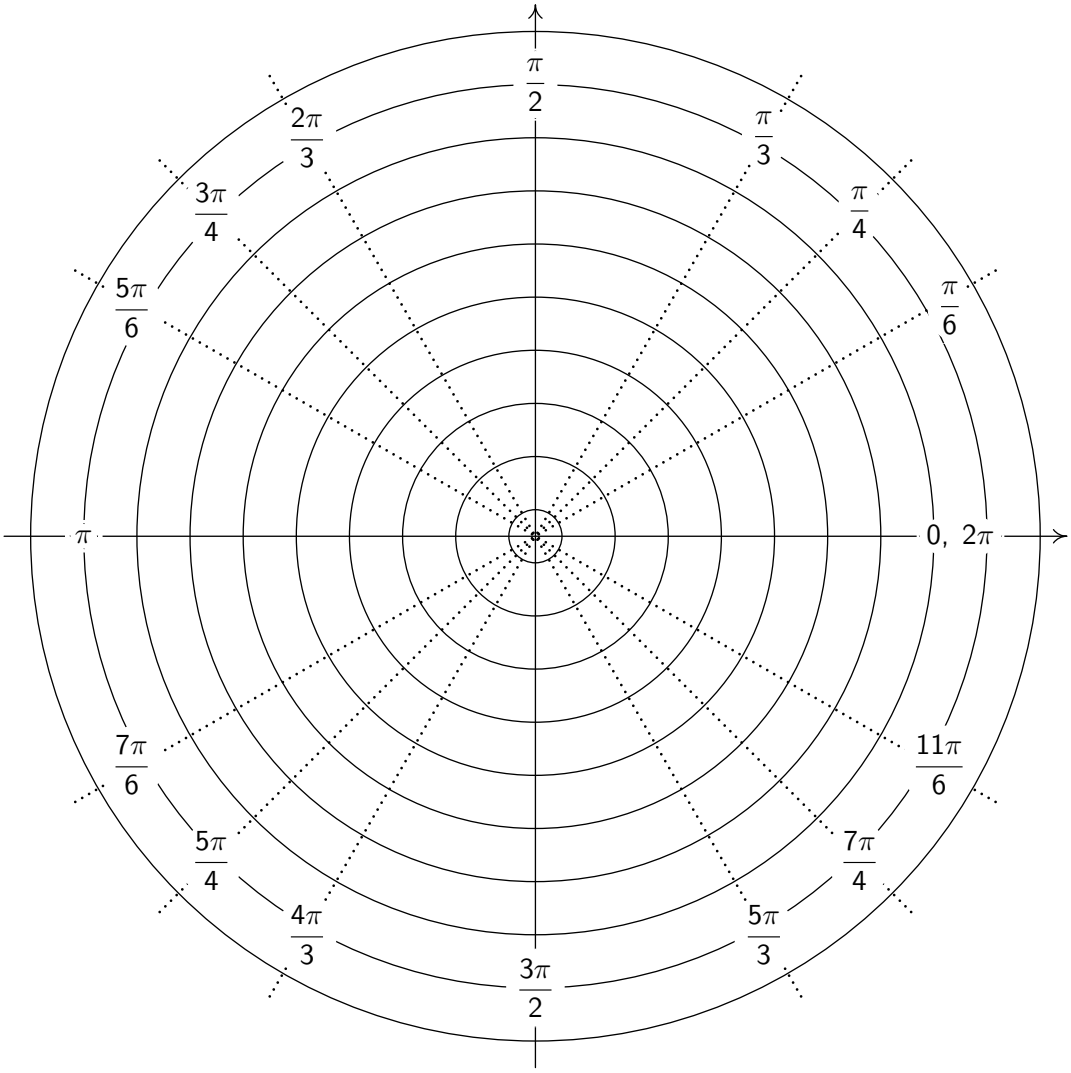
(b) Find an expression for $\dfrac{dy}{dx}$ in terms of t.

(c) Find the equations of both tangent lines to C at the origin.

6. Find the area under one arch of the cycloid: $\{x = t - \sin(t),\ y = 1 - \cos(t)$.

   Recall: $\cos^2(t) = \dfrac{1 + \cos(2t)}{2}$.

7. (a) Graph the 'rose' $r = 4\cos(2\theta)$ below. Be sure to label the tangents at the poles.



(b) Find the area of one leaf of this 'rose'. Recall: $\cos^2(t) = \dfrac{1+\cos(2t)}{2}$.