MATH 2600: TEST 02 (100 POINTS)

NAME: ____________________

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

1. Consider the integral: $\displaystyle\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$

   (a) Explain why this integral is improper.

   (b) Rewrite the improper integral as a limit of a proper integral.

   (c) Determine if the integral converges or diverges.

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2. For each of the given scenarios below, circle the most correct answer.

(a) If $\lim_{k\to\infty} a_k = 0$, then the series $\sum_{k=1}^{\infty} a_k$:

- definitely converges.
- definitely diverges.
- may or may not converge.

(b) If $a_k > \frac{1}{k^2}$, then the series $\sum_{k=1}^{\infty} a_k$:

- definitely converges.
- definitely diverges.
- may or may not converge.

(c) If $0 < a_k < \frac{1}{k^2}$, then the series $\sum_{k=1}^{\infty} a_k$:

- definitely converges.
- definitely diverges.
- may or may not converge.

(d) If $\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = 0$, then the series $\sum_{k=1}^{\infty} a_k$:

- definitely converges.
- definitely diverges.
- may or may not converge.

(e) If $\lim_{k\to\infty} \sqrt[k]{|a_k|} = 1$, then the series $\sum_{k=1}^{\infty} a_k$:

- definitely converges.
- definitely diverges.
- may or may not converge.

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3. Consider the series: $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+k}$.

(a) Find an explicit formula for the nth partial sum, S_n.

(b) Find $\displaystyle\lim_{n\to\infty} S_n$ and use this to determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+k}$ converges or diverges.

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4. (a) Consider the integral: $\displaystyle\int_1^{\infty} \frac{1}{x^2+1}\,dx$.

   i. Explain why this integral is improper.

   ii. Rewrite the improper integral as a limit of a proper integral.

   iii. Determine if the integral converges or diverges.

(b) Prove the series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+1}$ converges using the Integral Test.

HINT: Remember to check the conditions of the test apply!

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5. Determine whether the following series converge or diverge. Explain your reasoning.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{2k+1}{3-k}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{2^n}{n!}$

(c) $\displaystyle\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m^2}$

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6. (a) Does the series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ converge or diverge? Explain.

(b) Use $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ and the Limit Comparison Test to help you decide if $\sum_{k=1}^{\infty} \tan\left(\frac{1}{\sqrt{k}}\right)$ converges.

NOTE: Make sure you pass to a continuous variable, such as x, before applying L'Hopital's Rule ...

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7. Prove $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{3k+1}$ converges **conditionally**.

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8. Prove $\displaystyle\sum_{k=1}^{\infty} \frac{\sin(4k)}{k\sqrt{k}}$ converges **absolutely**.

HINT: $|\sin(\theta)| \leq 1 \ldots$