MATH 2500: PROJECT ALTERNATIVE (20 points.)

NAME: ____

1. Fill in the blanks below.

   (a) $f(a)$ gives you:

   - the ____________________ of the function with **input** $x = a$.

   - the ____________________-value on the graph of $y = f(x)$ at $x = a$.

   (b) $\lim_{x \to a} f(x)$ gives you:

   - what to ____________________ from the function when $x = a$.

   - the y-value which the y-values on the graph of $y = f(x)$ are ____________________ as $x \to a$.

   (c) If f is **continuous** at $x = a$ then $\lim_{x \to a} f(x) =$ ______________ .

   (d) If f is **differentiable** at $x = a$ then the graph of $y = f(x)$ near $(a, f(a))$ is **locally** ______________ .

   (e) $f'(a)$ gives you:

   - the ____ of outputs to inputs at $x = a$.

   - the ____ of the graph of $y = f(x)$ at $x = a$.

   (f) $\int_a^b f(x)\,dx$ gives the net ______________ between the graph of $y = f(x)$ and the x-axis.

   (g) $\int_a^b f'(x)\,dx$ gives the net ______________ in f over the interval $[a, b]$.

2. Suppose $C(x)$ represents the cost of producing x units, $R(x)$ represents the revenue generated by selling x units, and $P(x)$ represents the profit obtained by producing and selling x units.

(a) What is the relationship between C, R, and P?

(b) Use your answer to part (a) to write the relationship between the marginals: MC, MR and MP.

(c) Suppose P is differentiable on $(0, \infty)$ and $P(c)$ is a maximum.

i. What is $P'(c)$? Explain your reasoning.

ii. What is the relationship between $MC(c)$ and $MR(c)$ at this point?

3. Consider the limit: $\lim_{x \to 0} \frac{\sin(3x)}{2x}$.

(a) Explain why direct substitution results in an indeterminate form.

(b) Use the fact that $\lim_{u \to 0} \frac{\sin(u)}{u} = 1$ along with a substitution to find $\lim_{x \to 0} \frac{\sin(3x)}{2x}$.

(c) Interpret what your answer to part (b) means graphically.

4. Consider the limit: $\lim_{x\to\infty} \dfrac{\sin(3x)}{2x}$.

(a) Explain why we can't use direct substitution or the leading term test to determine this limit.

(b) Use the Squeeze Theorem to determine $\lim_{x\to\infty} \dfrac{\sin(3x)}{2x}$.

Be sure to write out each step of your analysis and explain your reasoning.

(c) Interpret what your work and your answer to part (b) means graphically.

HINT: $\dfrac{\sin(3x)}{2x} = \dfrac{1}{2x} \sin(3x) \ldots$

5. Let $f(x) = x^2 \ln(x)$

(a) Explain why the domain of f is $(0, \infty)$.

(b) Find the x-intercepts, if any.

(c) Why are there no y-intercepts?

(d) Explain why $\lim_{x \to 0^+} x^2 \ln(x)$ results in an indeterminate form.

(e) Use a table of values to approximate $\lim_{x \to 0^+} x^2 \ln(x)$.

Explain why your answer means the graph has a hole in the graph at $(0, 0)$.

(f) Find $\lim_{x \to \infty} x^2 \ln(x)$

(g) Find and simplify $f'(x)$ and make a sign diagram for $f'(x)$.

List the open intervals (if any) over which f is:

- **increasing:**
- **decreasing:**

List any local extrema (local maximums or minimums): (x, y):

(h) Find and simplify $f''(x)$ and make a sign diagram for $f''(x)$.

List the open intervals (if any) over which f is:

- **concave up:**
- **concave down:**

List any inflection points: (x, y):

(i) Sketch a detailed graph of $y = f(x)$ using the information you've gathered in parts (a) through (h).

Include the axis-intercepts, if any.

6. Let $f(x) = \dfrac{10}{1 + e^{-x}}$

(a) Explain why the domain of f is $(-\infty, \infty)$.

Why does this show the graph of $y = f(x)$ has no vertical asymptotes?

(b) Find the y-intercept of the graph of $y = f(x)$

(c) Why are there no x-intercepts?

(d) Find $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$ and use these to describe the end behavior of the graph of $y = f(x)$.

In particular, list any horizontal or slant asymptotes.

(e) i. Rewrite $f(x)$ so as to avoid the quotient rule when calculating $f'(x)$ and find $f'(x)$.

ii. Make a sign diagram for $f'(x)$ and explain why this shows f is always increasing.

(f) i. Find and simplify $f''(x)$. Write your final answer as a single fraction.

ii. Make a sign diagram for $f''(x)$.

List the intervals over which the graph of f is concave up or concave down.

- **concave up:**
- **concave down:**

iii. List any inflection points: (x, y):

(g) Sketch a detailed graph of $y = f(x)$ using the information you've gathered in parts (a) through (f).

7. On the first day of class, I wrote the 'equation': Limits + Precalculus = Calculus. In your own words, write a sentence, or two, on how we used the limit concept to take old familiar notions from Pre-calculus (College Algebra) and created something 'new' in Calculus for each of the two 'big' topics in Calculus:

   - the derivative:

   - the definite integral: