MATH 2500: TEST 04 (100 POINTS)

NAME: _____

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

1. Find and simplify the following derivatives:

   (a) For $f(x) = \sin^{-1}(5x)$, find $f''(x)$.

   (b) If $y = \tan^{-1}(5x)$, find $\dfrac{dy}{dx}$.

   (c) Find $\dfrac{d}{dx}[\ln(4 - x)]$.

   (d) If $F(x) = e^{-2x}$, find $F'(x)$.

2. Find the following integrals using the techniques demonstrated in class.

(a) $\int x\sqrt{x^2+9}\,dx$

(b) $\int \frac{1}{x^2+9}\,dx$

(c) $\int \frac{2x}{x^2+9}\,dx$

(d) $\int e^{\cos(x)}\sin(x)\,dx$

3. Find the equation of the line tangent to the graph of $f(x) = x \ln(x)$ at $(1, 0)$.

4. Find the area under the curve $y = \dfrac{x}{\sqrt{4 - x^2}}$ from $x = 0$ to $x = 1$.

5. Use the Fundamental Theorem of Calculus to find the indicated derivative:

(a) $D_x \left[\int_0^x \cos\left(t^2\right)\, dt \right]$

(b) $D_x \left[\int_0^{x^3} \cos\left(t^2\right)\, dt \right]$

6. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = x^{\ln(x)}$.

7. Let $P(t)$ represent the population of a particular species of bird in Lake County t years after 2020. Suppose

$$P'(t) = -5e^{-0.01t}, \quad t \geq 0$$

(a) Find $P'(0)$ and interpret what this number means in terms of birds and years.

(b) Find: $\int_0^{10} P'(t)\,dt$. Find an exact answer as well as an approximation rounded to the nearest inteteger.

(c) Interpret your answer to part (b) in terms of birds and years.

(d) If 500 birds were present in 2020, how many are left in 2030? Explain your reasoning.

8. Recall: $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$ and $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.

(a) Verify $\cosh^2(x) - \sinh^2(x) = 1$.

(b) Verify $D_x\left[\sinh(x)\right] = \cosh(x)$.

(c) Find $\displaystyle\int \frac{\cosh(x)}{\sqrt{1 - \sinh^2(x)}}\, dx$.

(d) **BONUS:** Find $\displaystyle\int \frac{\cosh(x)}{\sqrt{1 + \sinh^2(x)}}\, dx$.

BASIC INTEGRATION FORMULAS:

- $\int du = \int 1\, du = u + C$
- $\int u^p\, du = \dfrac{1}{p+1}\, u^{p+1} + C, \quad p \neq -1$
- $\int \sin(u)\, du = -\cos(u) + C$
- $\int \cos(u)\, du = \sin(u) + C$
- $\int \csc(u)\cot(u)\, du = -\csc(u) + C$
- $\int \sec(u)\tan(u)\, du = \sec(u) + C$
- $\int \csc^2(u)\, du = -\cot(u) + C$
- $\int \sec^2(u)\, du = \tan(u) + C$
- $\int \dfrac{1}{u}\, du = \ln|u| + C$
- $\int e^u\, du = e^u + C$

RECALL: If $a > 0$:

- $\int \dfrac{1}{\sqrt{a^2 - u^2}}\, du = \arcsin\left(\dfrac{u}{a}\right) + C = \sin^{-1}\left(\dfrac{u}{a}\right) + C$
- $\int \dfrac{1}{u^2 + a^2}\, du = \dfrac{1}{a}\arctan\left(\dfrac{u}{a}\right) + C = \dfrac{1}{a}\tan^{-1}\left(\dfrac{u}{a}\right) + C$
- $\int \dfrac{1}{u\sqrt{u^2 - a^2}}\, du = \dfrac{1}{a}\text{arcsec}\left(\dfrac{|u|}{a}\right) + C = \dfrac{1}{a}\sec^{-1}\left(\dfrac{|u|}{a}\right) + C$