MATH 2500: TAKE HOME 04 (20 POINTS)

DUE THE DAY OF TEST 4 AT THE BEGINNING OF CLASS

NAME: ____

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

SECTION 5.3 PRACTICE PROBLEMS

1. Use the Fundamental Theorem of Calculus to find $\int_{-\pi}^{\pi} \sin(x)\, dx$ and check your answer geometrically.

   FToC Solution:

   Geometric Check:

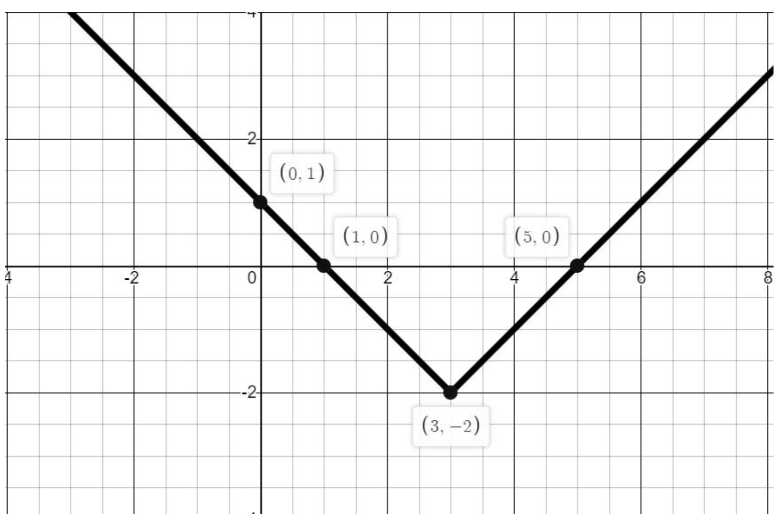
2. Find $\int_{-2}^{6} \frac{4-x}{2}\,dx$:

(a) by interpreting the integral as a net area:

(b) using the Fundamental Theorem of Calculus:

3. Use the Fundamental Theorem of Calculus to evaluate: $\displaystyle\int_1^4 \frac{3-2x}{4\sqrt{x}}\,dx$

4. The graph of $y = f(t)$ is below. Let $F(x) = \displaystyle\int_0^x f(t)\,dt$ for $0 \le x \le 8$



(a) Find $F(5)$.

(b) List the open interval(s) in $(0, 8)$ where F is increasing.

5. Use the Fundamental Theorem of Calculus to help you find the following derivatives:

(a) $D_x\left[\int_0^x \sqrt{t^3+1}\,dt\right]$

(b) $D_x\left[\int_0^{x^2} \sqrt{t^3+1}\,dt\right]$

(c) $D_x\left[\int_0^{\cos(x)} \sqrt{t^3+1}\,dt\right]$

(d) $D_x\left[\int_{x^2}^{\cos(x)} \sqrt{t^3+1}\,dt\right]$

SECTION 5.4 PRACTICE PROBLEMS

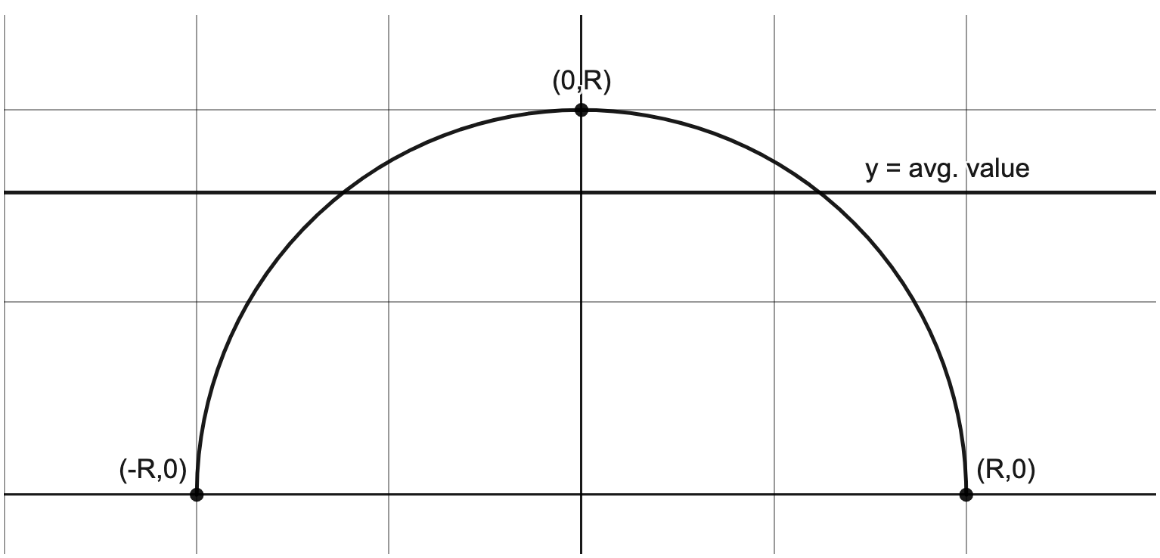
1. Let $f(x) = \sqrt{R^2 - x^2}$ for $R > 0$.

   (a) Show that the graph of f is the top half of the circle $x^2 + y^2 = R^2$.

   (b) Find the average value of f over $[-R, R]$.

   HINT: You'll need to interpret the integral as an area ...

   (c) Interpret your answer to part (b) geometrically on the graph below:



SECTION 5.5 PRACTICE PROBLEMS

1. $\displaystyle\int \frac{2}{(1-2x)^3}\,dx$

2. $\displaystyle\int \frac{\cos(\theta)}{\sqrt{1+\sin(\theta)}}\,d\theta$

3. Find the following definite integral: $\displaystyle\int_0^4 \frac{4x}{\sqrt{2x+1}}\,dx$.

4. (a) Find $\displaystyle\left|\int_0^{\pi} \cos(x)\,dx\right|$ and $\displaystyle\int_0^{\pi} |\cos(x)|\,dx$.

(b) In general, is it true that $\displaystyle\left|\int_a^b f(x)\,dx\right| = \int_a^b |f(x)|\,dx$?

(c) Under what circumstances would $\displaystyle\left|\int_a^b f(x)\,dx\right| = \int_a^b |f(x)|\,dx$?

EXTRA (EXTRA!) PRACTICE WITH SUBSTITUTION

1. $\int \sqrt{1-x}\, dx$

2. $\int x\sqrt{1-x^2}\, dx$

3. $\int x\sqrt{1-x}\, dx$

4. $\int \sin(x)\sqrt{1-\cos(x)}\, dx$

5. $\int \frac{3}{(x-1)^3}\, dx$

6. $\int \frac{3x}{(x-1)^5}\, dx$

7. $\int \frac{3x}{(x^2-1)^2}\, dx$

8. $\int \frac{3}{x^2-2x+1}\, dx$

   HINT: Factor...

9. $\int \frac{\sec(x)}{\tan^2(x)+1}\, dx$

   HINT: Identities...

10. $\int_0^{2\pi} \sqrt{1-\cos(x)}\, dx$

    HINT: $1-\cos(x) = 2\sin^2(x/2)\ \ldots$

EXTRA (EXTRA!) PRACTICE WITH SUBSTITUTION - ANSWERS

1. $\int \sqrt{1-x}\,dx = -\frac{2}{3}(1-x)^{3/2} + C$

2. $\int x\sqrt{1-x^2}\,dx = -\frac{1}{3}(1-x^2)^{3/2} + C$

3. $\int x\sqrt{1-x}\,dx = \frac{2}{5}(1-x)^{5/2} - \frac{2}{3}(1-x)^{3/2} + C$

4. $\int \sin(x)\sqrt{1-\cos(x)}\,dx = \frac{2}{3}(1-\cos(x))^{3/2} + C$

5. $\int \frac{3}{(x-1)^3}\,dx = -\frac{3}{2}(x-1)^{-2} + C$

6. $\int \frac{3x}{(x-1)^5}\,dx = -(x-1)^{-3} - \frac{3}{4}(x-1)^{-4} + C$

7. $\int \frac{3x}{(x^2-1)^2}\,dx = -\frac{3}{2}(x^2-1)^{-1} + C$

8. $\int \frac{3}{x^2-2x+1}\,dx = \int \frac{3}{(x-1)^2}\,dx = -3(x-1)^{-1} + C$

9. $\int \frac{\sec(x)}{\tan^2(x)+1}\,dx = \int \frac{\sec(x)}{\sec^2(x)}\,dx = \int \cos(x)\,dx = \sin(x) + C$

10. $\int_0^{2\pi} \sqrt{1-\cos(x)}\,dx = \int_0^{2\pi} \sqrt{2\sin^2(x/2)}\,dx = \ldots = 4\sqrt{2}$

SECTION 7.1 / 7.5 PRACTICE PROBLEMS

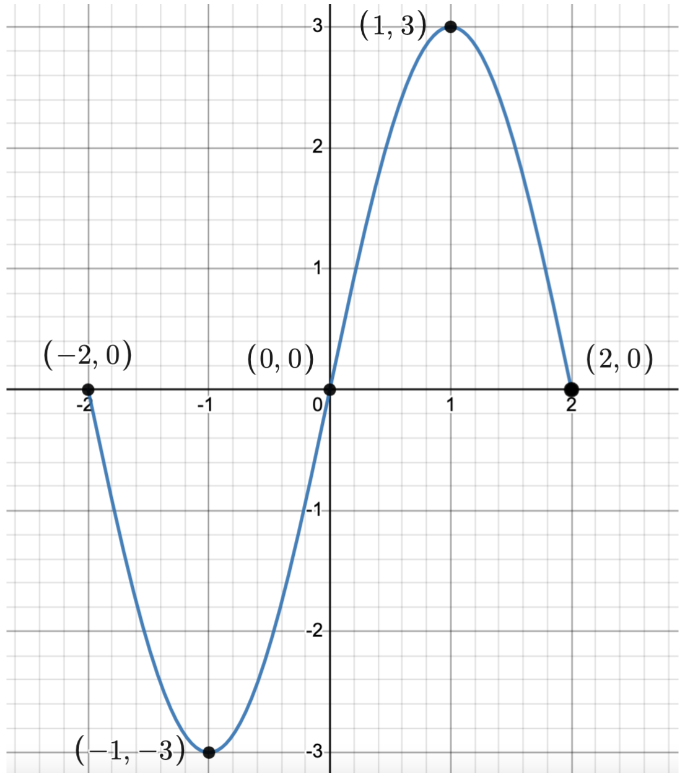
1. Suppose f is one-to-one and $(-1, 3)$ is on the graph of f. If $f'(-1) = 5$, find:

   (a) the equation of the tangent line to the graph of $y = f(x)$ when $x = -1$.

   (b) the equation of the tangent line to the graph of $y = f^{-1}(x)$ when $x = 3$.

2. Find and simplify $\frac{d^2y}{dx^2}$ if $y = \sin^{-1}(2x)$.

3. Consider the graph of $y = S(x)$ below.



(a) List the domain and range of S using interval notation.

Domain: ____________________ Range: ____________________

(b) List the minimum and maximum of S if they exist.

Minimum: ____________________ Maximum: ____________________

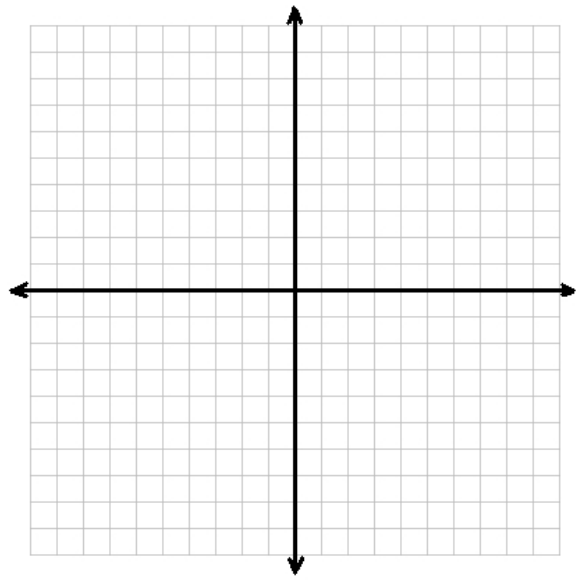
(c) Determine where:

$S'(x) > 0$: _____________ $S'(x) < 0$: _____________ $S'(x) = 0$: _____________

(d) Estimate $S'(0)$. Explain your reasoning using the graph.

(e) Solve $S(x) = 0$ and explain how this shows that S is not one-to-one.

(f) Let R be S but with the restricted domain of $-1 \leq x \leq 1$. Graph $y = R(x)$ below.



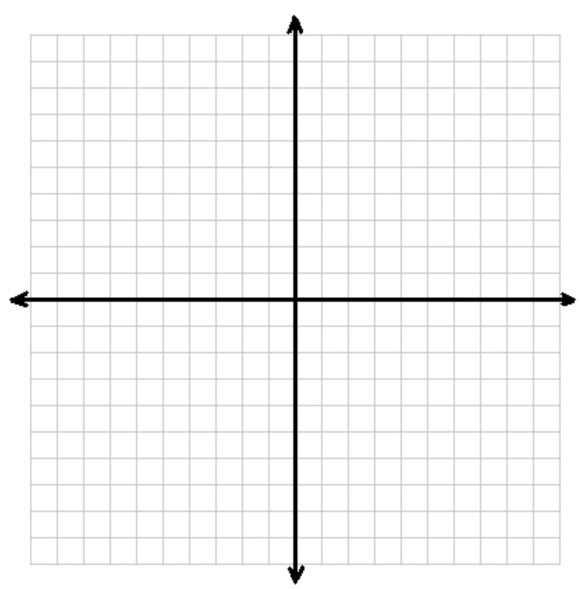
(g) List the domain and range of R using interval notation.

Domain: ____________________ Range: ____________________

(h) List the minimum and maximum of R if they exist.

Minimum: ____________________ Maximum: ____________________

(i) Explain why R is one-to-one and sketch a careful graph $y = R^{-1}(x)$ below.



(j) Estimate $y = \left(R^{-1}\right)'(0)$ using your graph. How does this compare with $S'(0)$?

4. Find the following antiderivatives using the techniques demonstrated in class.

   Check your answer by taking the derivative.

   (a) $\displaystyle\int \frac{1}{\sqrt{7-x^2}}\,dx$

   (b) $\displaystyle\int \frac{1}{8x^2+1}\,dx$

   (c) $\displaystyle\int \frac{1}{x\sqrt{25x^2-49}}\,dx$

5. Find the following definite integrals:

   (a) $\displaystyle\int_0^{\frac{3}{2}} \frac{1}{\sqrt{9-x^2}}\,dx$

   (b) $\displaystyle\int_{-2}^{2} \frac{1}{x^2+4}\,dx$

6. Find the equation of the line tangent to the graph of $f(x) = \tan^{-1}\left(\sqrt[5]{x}\right)$ when $x = -1$.

7. Find the net area between the curve $y = \dfrac{1}{x\sqrt{x^2 - 9}}$ and the x-axis over the interval $\left[3\sqrt{2}, 6\right]$.

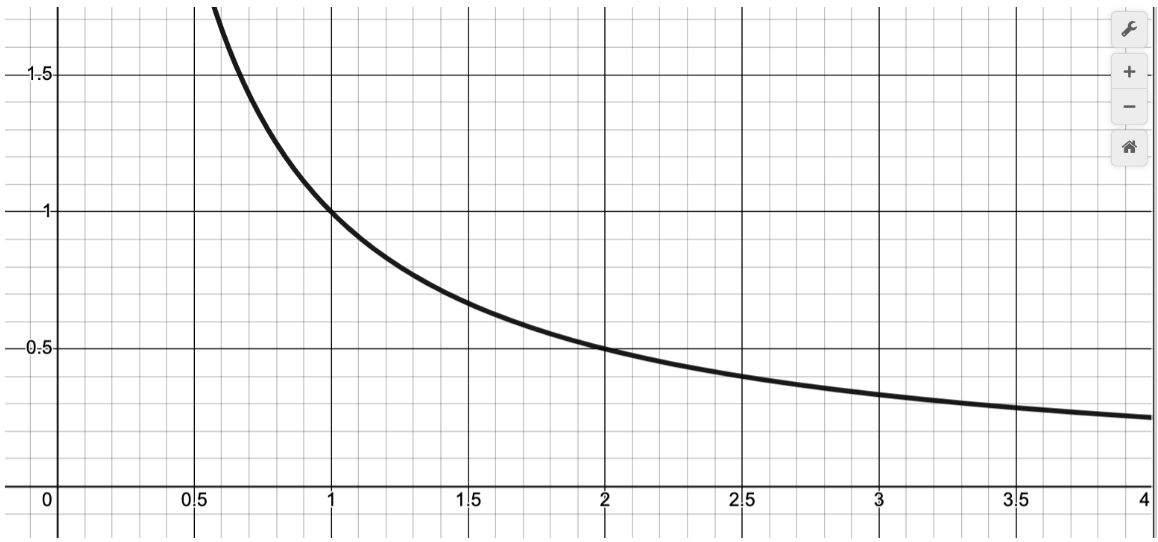
8. Find the following integral using the techniques demonstrated in class: $\displaystyle\int \frac{1}{\sqrt{8x - x^2}}\,dx$

   HINT: Complete the Square ...

SECTION 7.2 / 7.3 PRACTICE PROBLEMS

1. (a) State the integral definition of $\ln(x)$: for $x > 0$, $\ln(x) =$

   (b) On the graph below, shade the area equal to $\ln(2)$ and explain geometrically $0.5 < \ln(2) < 1$.



2. Let $f(x) = x^{3\cos(x)}$.

(a) Explain why we can't use the power rule: $D_x\left[x^k\right] = k\,x^{k-1}$ to find $f'(x)$.

(b) Explain why we can't use the exponential rule: $D_x\left[a^u\right] = a^u\,ln(a)\,u'$ to find $f'(x)$.

(c) Find $f'(x)$ using logarithmic differentiation. Be sure to write your final answer in terms of 'x' only.

3. Find and simplify the indicated derivative. Use Desmos to check your answers graphically!

(a) $A'(t)$ if $A(t) = 5000\, e^{-0.01t}$

(b) $D_t\left[\dfrac{500}{1+24e^{-0.01t}}\right]$

(c) $\dfrac{dy}{dx}$ if $y = \ln\sqrt{2x+1}$

(d) $\dfrac{d}{d\theta}\left[\ln|\sin(\theta)|\right]$

(e) $H'(t)$ if $H(t) = \dfrac{e^t - e^{-t}}{e^t + e^{-t}}$

(f) $F''(\theta)$ if $F(\theta) = \ln|\sec(\theta) + \tan(\theta)|$

(g) $\dfrac{dF}{ds}$ if $F(s) = \dfrac{e^{-2s}}{s^2+1}$

(h) $y^{(5)}$ if $y = \dfrac{e^t + e^{-t}}{2}$

(i) y' if $y = x\,\sin^{-1}(x) + \sqrt{1-x^2}$

(j) $\dfrac{d^2y}{dx^2}$ if $y = \sin^{-1}(4x)$

(k) y' if $y = x\,\tan^{-1}(x) - \dfrac{1}{2}\ln\left(x^2+1\right)$

(l) $g'(x)$ if $g(x) = \sec^{-1}\left(x^2\right)$

4. Find the following antiderivatives using the techniques demonstrated in class.

Check your answer by taking the derivative.

(a) $\displaystyle\int \frac{1}{2x-5}\,dx$

(b) $\displaystyle\int x^2\, e^{x^3}\,dx$

(c) $\displaystyle\int \frac{e^{2x}+9}{e^{2x}}\,dx$

(d) $\displaystyle\int \frac{e^{2x}}{e^{2x}+9}\,dx$

(e) $\displaystyle\int \frac{e^{x}}{e^{2x}+9}\,dx$

(f) $\displaystyle\int \frac{\cos(x)}{1-\sin(x)}\,dx$

(g) $\displaystyle\int \frac{1}{x\sqrt{9-[\ln(x)]^2}}\,dx$

SECTION 6.1 / 8.8 PRACTICE PROBLEMS

1. Let $A(t)$ represent the amount of an investment in dollars t t years after an account opened. Suppose:

$$A'(t) = 1000e^{0.025t}, \quad t \geq 0$$

(a) Find $A'(0)$ and interpret what this number means in terms of time and money.

(b) Find: $\int_0^5 A'(t)\,dt$. Find an exact answer as well as an approximation rounded to two decimal places.

(c) Interpret your answer to part (b) in terms of time and money in the account.

(d) If \$40,000 was initially invested in the account how much money is in the account after 5 years? Round your answer to the nearest cent.

2. Suppose $p(t)$ represents the population of a certain species of fish t years after 2020.

(a) Suppose $\int_0^3 p'(t)\,dt = -500$. What does this mean in terms of the fish population?

(b) Suppose $\int_0^3 |p'(t)|\,dt = 2500$. What does this mean in terms of the fish population?

(c) What was the total increase in fish population between 2020 and 2023? Total decrease?

3. Below is a list of hourly temperature data for April 21st, 2023 measured at Burke Lakefront Airport:

Time	Temperature in $^\circ$ F
5:51 AM	65° F
6:51 AM	63° F
7:51 AM	64° F
8:51 AM	67° F
9:51 AM	68° F
10:51 AM	71° F
11:51 AM	72° F

(a) Average these seven temperature values to find an estimate of the average temperature on this day.

(b) Assuming the temperature function, T is continuous, use Simpson's Rule to help you approximate the average temperature during this period. Write out the sum then use desmos for the calculations.

NOTE: We're looking for the average value, $\overline{T}$ of the continuous function $T(t)$ ...