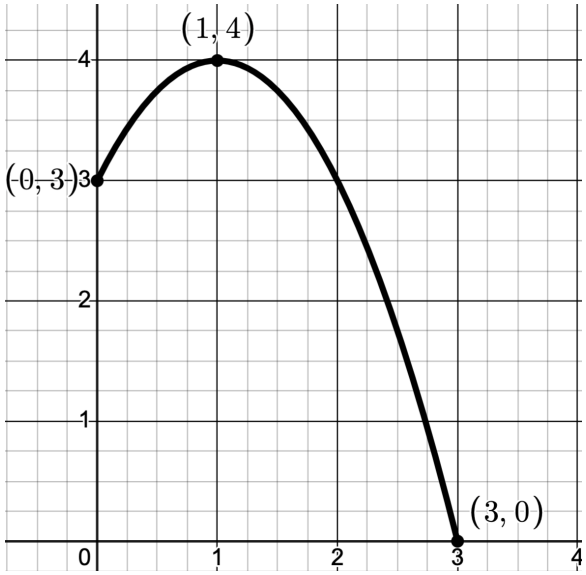
MATH 2500: TEST 03 IN CLASS PORTION (85 POINTS)

NAME: _____

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

1. The complete graph of $y = f(x)$ is below. Assume f is continuous on $[0, 3]$ and differentiable on $(0, 3)$.



(a) Find the values guaranteed by the Extreme Value Theorem (EVT).

(b) Find all values guaranteed by Fermat's Theorem.

(c) Graphically approximate all values guaranteed by the Mean Value Theorem.

(d) List the open intervals (if any) over which:

$f'(x) > 0$: $f'(x) < 0$:

$f''(x) > 0$: $f''(x) < 0$:

2. Let $f(x) = \dfrac{10x}{x^2+1}$.

(a) Explain why the domain of f is $(-\infty, \infty)$.

(b) Find the x- and y-intercepts of the graph of $y = f(x)$.

(c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ and interpret your results graphically.

(d) It turns out that $f'(x) = \dfrac{10 - 10x^2}{(x^2+1)^2}$

i. Make a Sign Diagram for $f'(x)$

ii. Interpret your Sign Diagram for $f'(x)$ to determine:

Interval(s) over which f is increasing:

Interval(s) over which f is decreasing:

Local maximum(s): $(c, f(c))$

Local minumum(s): $(c, f(c))$

(e) It turns out that $f''(x) = \dfrac{20x^3 - 60x}{(x^2+1)^3}$

i. Make a Sign Diagram for $f''(x)$

ii. Interpret your Sign Diagram for $f''(x)$ to determine:

Interval(s) over which f is concave up:

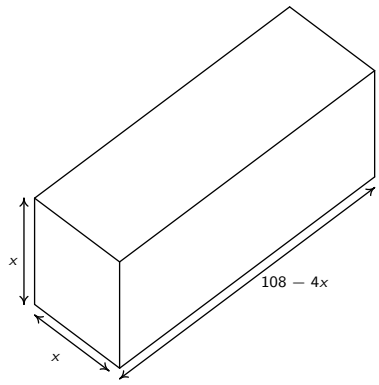
Interval(s) over which f is concave down:

Inflection point(s): $(c, f(c))$:

(f) Use the information you've gathered in parts (a) through (e) to sketch the graph of $y = f(x)$ below.

Label all key points.

3. A rectangular box with square base is to be shipped via US Mail using 'Parcel Post.' The dimensions of the box (in inches) are restricted so that the relationship between the dimensions are as depicted below.



(a) Show the volume of the box as a function of x is: $V(x) = x^2(108 - 4x)$.

(b) What is a reasonable applied domain here? Explain your reasoning.

(c) Use Calculus to find the dimensions of the box with the maximum volume. What is the maximum volume? Be sure to explain how you know you've found the maximum.

4. Recall the surface area of a sphere of radius r is $S = 4\pi r^2$.

   (a) Find an expression for $\frac{dS}{S}$.

   (b) Find and interpret $\frac{dS}{S}$ with $r = 1$ cm and $dr = \pm 0.1$ cm.

5. The velocity of an object traveling up and down the y-axis is given by $v(t) = 2\sin(t)$.

   If the object is initially at $(0, 5)$, find and expression for the position, $s(t)$.

6. Find the following antiderivatives:

(a) $\displaystyle\int \frac{2x+1}{4\sqrt{x}}\,dx$

(b) $\displaystyle\int \sec(2\theta)\left[\tan(2\theta)+\sec(2\theta)\right]\,d\theta$

MATH 2500: TEST 03 TAKE HOME PORTION (15 POINTS)

NAME: _____

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

You may use your notes and desmos - but that's it!

1. (a) Find $\int_{-1}^{1} (2x - 1)\, dx$ by evaluating the limit of a right endpoint sum.

(b) Check your answer to part (a) by interpreting $\int_{-1}^{1} (2x - 1)\, dx$ as a difference of areas.

Include a graph and explain your reasoning.