MATH 2500: TAKE HOME 03 (20 POINTS)

DUE THE DAY OF TEST 3 AT THE BEGINNING OF CLASS

NAME: ____

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

SECTION 4.1 PRACTICE PROBLEMS

1. Let $f(x) = \cos(2x) - 2\sin(x)$.

   (a) Explain why f satisfies the conditions of the Extreme Value Theorem (EVT) on the interval $[0, \pi]$.

   (b) Analytically find all values guaranteed by the EVT.
   Check your answer using desmos.

SECTION 4.2 PRACTICE PROBLEMS

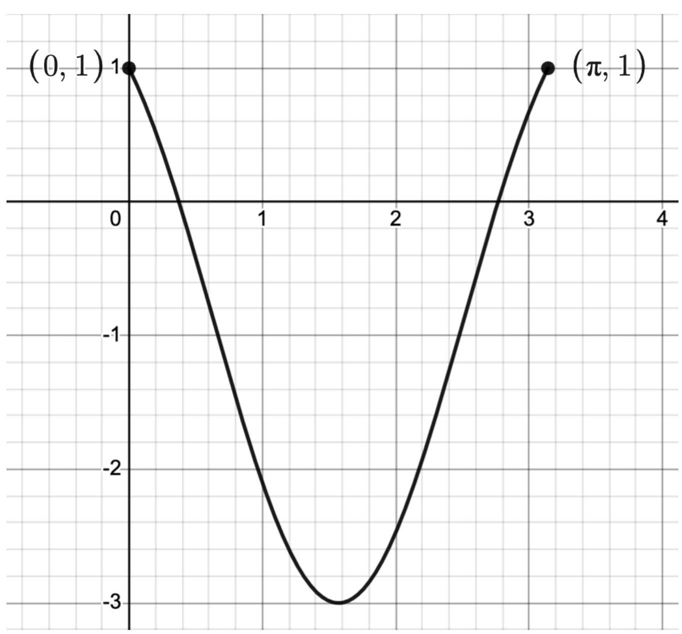
1. Let $f(x) = \cos(2x) - 2\sin(x)$.

   NOTE: This is the same function as in #1.

   (a) Explain why f satisfies the conditions of the Mean Value Theorem (MVT) on the interval $[0, \pi]$.

   (b) Analytically find all values guaranteed by the MVT. (Feel free to re-use your work from #1.)

   (c) Interpret your answers to part (b) graphically in terms of slopes on the graph below:



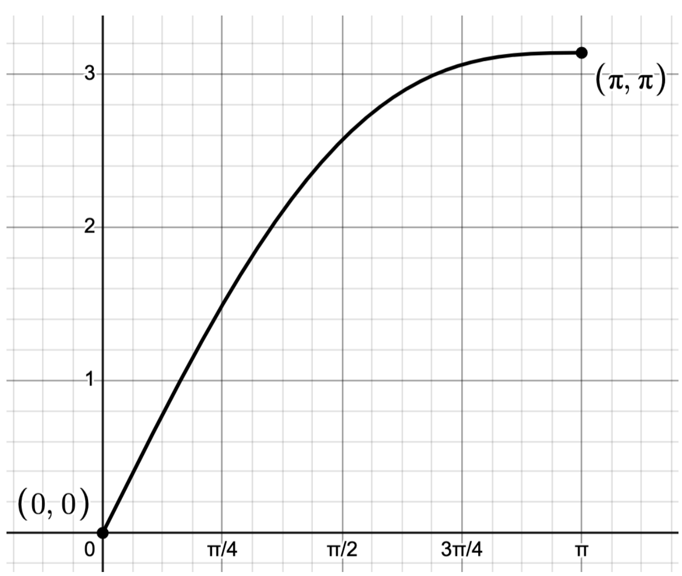
   (d) What other theorem regarding slopes applies in this scenario? Explain.

2. Let $f(x) = x + \sin(x)$.

(a) Explain why f satisfies the conditions of the Mean Value Theorem (MVT) on the interval $[0, \pi]$.

(b) Analytically find all values guaranteed by the MVT.

(c) Interpret your answers to part (b) graphically in terms of slopes on the graph below:



3. Suppose that on October 9th, it was 45°F at 8 AM, 53°F at Noon, and 45°F again at 4 PM. If the temperature throughout the day is a continuous function of time, which theorem: Intermediate Value Theorem (IVT), Extreme Value Theorem (EVT), Mean Value Theorem (MVT), or Rolle's Theorem (RT) could be used to justify each of the conclusions below?

   (a) On October 9th, there was a high and low temperature.

   (b) At some time between 8 AM and Noon it was precisely 47.3°F.

   (c) At some time between Noon and 4 PM the temperature was falling at exactly 2°F per hour.

   (d) At some time between 8 AM and 4 PM the temperature was neither rising nor falling.

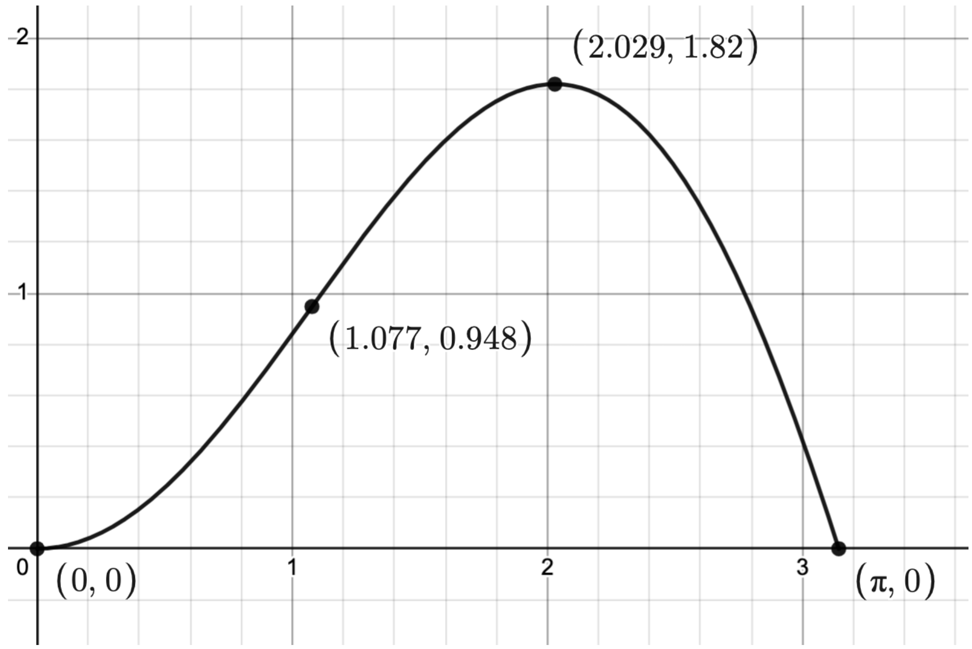
   HINT: There are two answers to this one!

SECTION 4.3 and 4.4 PRACTICE PROBLEMS

1. The complete graph of $y = f(x)$ is shown below. Use the graph to answer the following questions.

   BE SURE TO USE THE GRAPH TO <u>EXPLAIN</u> YOUR REASONING!

   NOTE: You may assume $(2.029, 1.82)$ is a local maximum and that $(1.077, 0.948)$ is an inflection point.



   (a) Determine the x-values where:

   $f(x) = 0$:

   $f'(x) = 0$:

   (b) List the open intervals over which:

   $f(x) > 0$:

   $f(x) < 0$:

   $f'(x) > 0$:

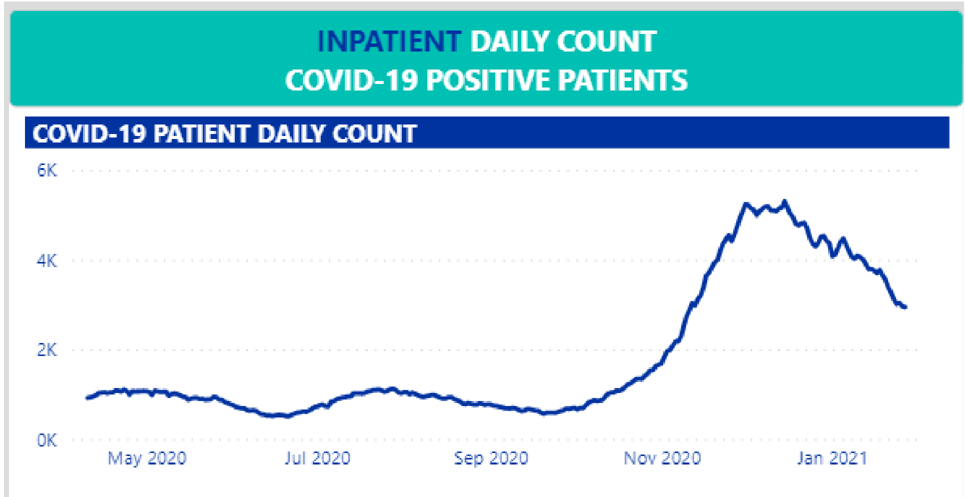
   $f'(x) < 0$:

   $f''(x) > 0$:

   $f''(x) < 0$:

   (c) Assuming f satisfies the conditions of Rolle's Theorem, find all values c guaranteed by Rolle's Theorem.

2. The graph below was taken from https://ohiohospitals.org/covid19data on January 28th, 2021:



On the graph, highlight and label one segment which (roughly) represents the following scenarios.

For brevity, we'll use 'patients' to mean 'inpatient COVID positive patients' and 'the rate of change' to mean 'the rate of change of inpatient COVID positive patients with respect to time.'

NOTE: There are several correct answers - you just need to highlight **one** segment for **each** scenario.

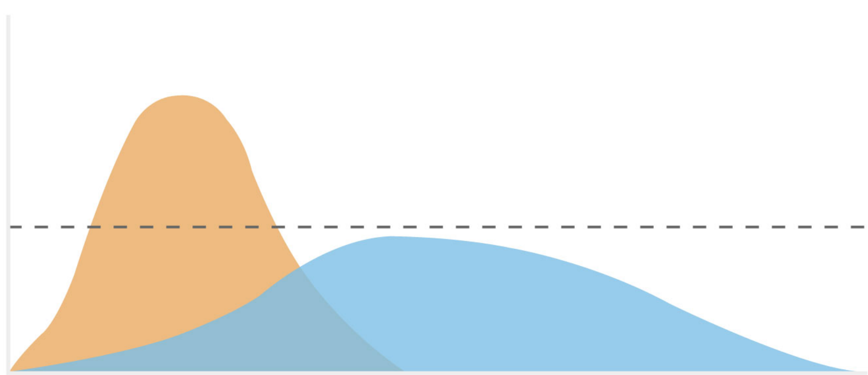
(a) The number of patients is **decreasing** and the rate of change is **decreasing**. (Label this 'a.')

(b) The number of patients is **decreasing** and the rate of change is **increasing**. (Label this 'b.')

(c) The number of patients is **increasing** and the rate of change is **increasing**. (Label this 'c.')

(d) The number of patients is **increasing** and the rate of change is **decreasing**. (Label this 'd.')

(e) The phase 'flatten the curve' along with the graphic below was often shown during the pandemic:



What does 'flattening the curve' mean in terms of first and/or second derivatives?

3. Sketch the graph of a function f whose information is given below:

- f is continuous on $(-\infty, \infty)$.
- $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$

$(+)$	0	$(-)$	0	$(-)$
	-1		5	

A sign diagram for $f(x)$

$(-)$	$?$	$(-)$	0	$(+)$	$?$	$(-)$
	-1		1		5	

A sign diagram for $f'(x)$

$(-)$	$?$	$(+)$	$?$	$(+)$
	-1		5	

A sign diagram for $f''(x)$

4. Let $f(x) = \dfrac{5x}{\sqrt{x^2+1}}$.

(a) Using the properties of continuity from Section 2.6, explain why f is continuous for all real numbers.

(b) Find the x- and y-intercepts of the graph.

(c) Use the leading term test (Section 2.5) to help you find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$:

- $\lim_{x\to-\infty} f(x) =$

- $\lim_{x\to\infty} f(x) =$

Use your answers to find the horizontal asymptotes of the graph of $y = f(x)$:

(d) i. Find and simplify $f'(x)$.

ii. Make a Sign Diagram for $f'(x)$.

iii. Use your Sign Diagram for $f'(x)$ in order to prove f is always increasing.

(e) i. Find and simplify $f''(x)$.

ii. Make a Sign Diagram for $f''(x)$.

iii. Interpret your Sign Diagram for $f''(x)$ in order to determine the open intervals over which f is:

concave up:

concave down:

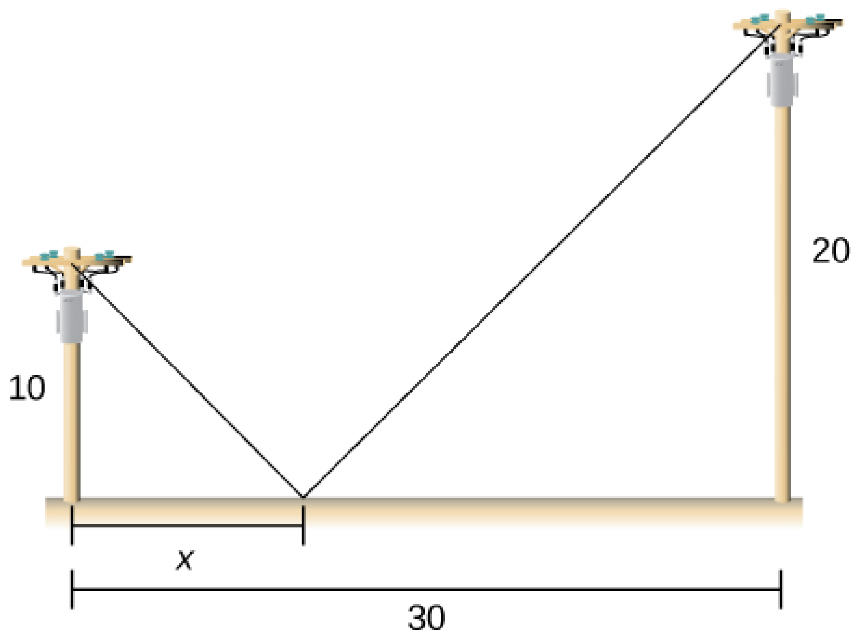
locations of any inflection points: $(c, f(c))$:

(f) Use your results from parts (a) through (e) to sketch an incredibly detailed graph of $y = f(x)$.

Label the intercepts, asymptotes, any local extrema and inflection points.

SECTION 4.5 PRACTICE PROBLEMS

1. Two poles are connected by a wire that is also connected to the ground between the poles. The first pole is 10 ft. tall and the second pole is 20 ft. tall. There is a distance of 30 ft. between the two poles. Let 'x' denote the distance, in feet, the wire is anchored from the 10 ft. tall pole.[1]



   (a) Assuming the wire is taut, show the length of the wire L is $L(x) = \sqrt{x^2 + 100} + \sqrt{(30 - x)^2 + 400}$.

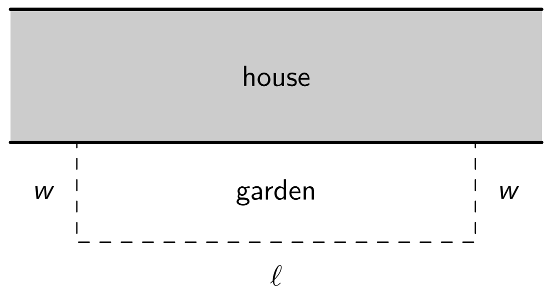
   What are the restrictions on x?

[1] Taken from OpenStax Calculus Volume 1

(b) Find the value of x which minimizes L using a sign diagram for $L'(x)$.

(c) What is the minimum length of wire required?

2. Taylor wants to plant a 200 square foot rectangular garden along the side of her home. Since the garden will be against the house, she has no need to fence along that side of the garden.



(a) Find a formula for the amount of fencing required F in terms of just ℓ, $F(\ell)$.

What restrictions are there on ℓ?

HINT: The garden is to be 200 square feet, so you can use this to help relate w and ℓ ...

(b) Find the critical number(s) for F and identify the one which lies in the applied domain.

(c) Use the second derivative test to prove the critical number you found in (b) produces a local minimum.

(d) How can you conclude that the local minimum in this case is the absolute minimum?

(e) What is the minimum amount of fencing required for a 200 square foot garden in this scenario?

What are the dimensions of the garden in this case?

SECTION 4.6 PRACTICE PROBLEMS

1. Let $y = f(x) = \sqrt{1+x}$.

   (a) Find an expression for linearization of f at $x = 0$, $L_0(x)$. Check your answer using desmos.

   (b) Use $L_0(x)$ to approximate $\sqrt{0.96}$.

   (c) Find an expression for dy.

   (d) Use differentials to approximate $\sqrt{0.96}$ using $x = 0$ and $\Delta x = dx = -0.04$.

2. The volume of a sphere V of radius r is given by: $V = \frac{4\pi}{3} r^3$.

   (a) Find an expression for dV.

   (b) Suppose the radius of a sphere is measured as 1 ± 0.1 centimeter.

      i. Find the volume of the sphere using $r = 1$ centimeter. Don't forget the units on your answer.

      ii. Calculate dV with $r = 1$ and $dr = \pm 0.1$. What does your answer represent?

      iii. Calculate $\frac{dV}{V}$. What does your answer represent?

SECTION 4.8 PRACTICE PROBLEMS

1. **EXPLORATION:** Consider $f(x) = \cos(x)$ on the interval $\left[0, \frac{\pi}{2}\right]$.

   (a) Explain why $f(x) = \cos(x)$ satisfies the conditions of the fixed point theorem[2] on $\left[0, \frac{\pi}{2}\right]$..

   (b) Write the equation you need to solve to find the fixed point:

   (c) Rewrite the equation in the form $F(x) = 0$ and use Newton's Method to approximate the fixed point.

   Equation: $F(x) =$ ______________________________________ $= 0$

   Use $x_0 = 0$ and iterate until you have found the answer accurate to five decimal places.

   Record your iterates (e.g., x_1, x_2, etc.) below and explain how you know when to stop iterating.

[2] Refer to Section 2.6 Practice Problems

SECTION 4.9 PRACTICE PROBLEMS

1. (a) Find and simplify $D_x[x\sin(x) + \cos(x)]$

   (b) Use part (a) to help you find $\int x\cos(x)\,dx$.

2. Find the following antiderivatives. Check your answers by taking derivatives.

(a) $\displaystyle\int \frac{4}{3\sqrt{x}}\,dx$

(b) $\displaystyle\int 2x^3(1-x)\,dx$

(c) $\displaystyle\int \frac{1-x}{2x^3}\,dx$

(d) $\displaystyle\int \left[\cos(5\theta) - \sin\left(\frac{\theta}{5}\right)\right]\,d\theta$

(e) $\displaystyle\int \sec(4t)\left[\tan(4t) - \sec(4t)\right]\,dt$

3. The acceleration of a particle moving up and down the y-axis is given by $a(t) = -8\sin(2t)$.

(a) If the initial velocity of the particle is 12, find an expression for the velocity function, $v(t)$.

(b) If the particle is initially at $y = -2$, find a formula for the position of the object $s(t)$.

(c) i. Where is the particle when $t = \frac{\pi}{2}$?

ii. In which direction is the particle moving when $t = \frac{\pi}{2}$?

iii. Find the acceleration of the particle when $t = \frac{\pi}{2}$.

Interpret what this means in terms of the velocity and speed of the particle.

SECTION 5.1 PRACTICE PROBLEMS

1. Find the area under the graphs of the following functions by using the limit of a right endpoint sum.

   NOTE: Here are some formulas which may or may not be helpful:

   - $\displaystyle\sum_{i=1}^{n} c\,A_i = c\sum_{i=1}^{n} A_i$
   - $\displaystyle\sum_{i=1}^{n} A_i \pm B_i = \sum_{i=1}^{n} A_i \pm \sum_{i=1}^{n} B_i$
   - $\displaystyle\sum_{i=1}^{n} c = cn$
   - $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
   - $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
   - $\displaystyle\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

   (a) $f(x) = 4 - x$ on $[0, 4]$.

   NOTE: You can check this one geometrically!

(b) $f(x) = 2x - x^2$ on $[0, 2]$.

(c) $f(x) = x^2 + 2x + 1$ on $[-2, 1]$.

SECTION 5.2 PRACTICE PROBLEMS

1. Find the following integrals by interpreting them in terms of areas.

   Include a sketch to explain your reasoning.

   (a) $\displaystyle\int_{-1}^{4} (1 - x)\, dx$

   (b) $\displaystyle\int_{-1}^{4} |1 - x|\, dx$

   (c) $\displaystyle\int_{0}^{6} \sqrt{6x - x^2}\, dx$

2. Let $f(x) = 4x\sqrt{x^2+1}$.

(a) Show $f(-x) = -f(x)$ for all x.

(b) Use part (a) to help you find $\int_{-5}^{5} f(x)\,dx$. Explain your reasoning.

3. Find $\displaystyle\int_{-1}^{2} \left(3x - x^2\right)\, dx$ by finding the limit of a right endpoint sum.

NOTE: Here are some formulas which may or may not be helpful:

- $\displaystyle\sum_{i=1}^{n} c\, A_i = c \sum_{i=1}^{n} A_i$
- $\displaystyle\sum_{i=1}^{n} A_i \pm B_i = \sum_{i=1}^{n} A_i \pm \sum_{i=1}^{n} B_i$
- $\displaystyle\sum_{i=1}^{n} c = cn$
- $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\displaystyle\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$