MATH 2500: TEST 01 (100 POINTS)

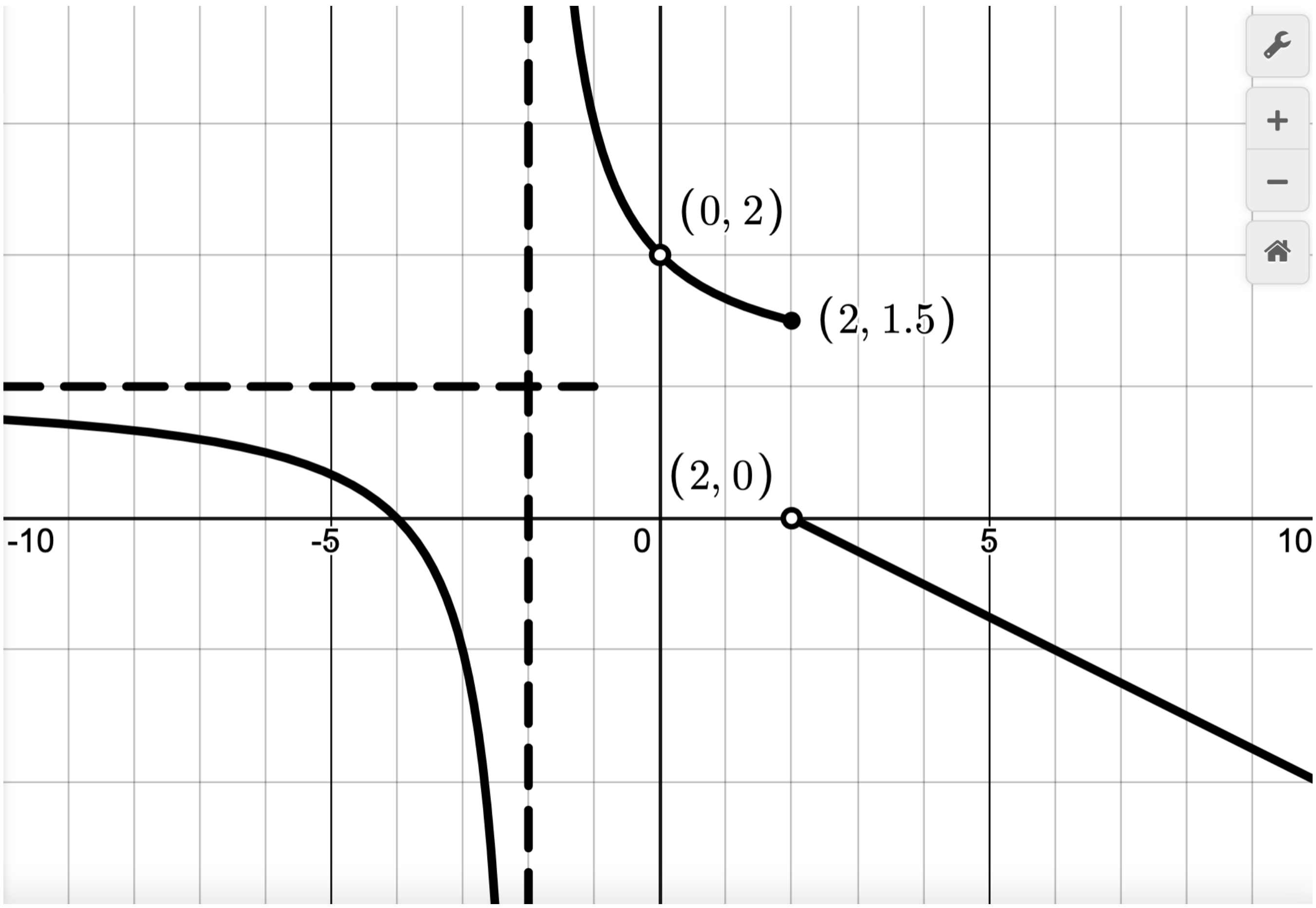
NAME: ______________________________

DIRECTIONS: Make sure your work is neat and complete and uses the techniques demonstrated in class.

1. Consider the complete graph of the function f below. Use the graph to find the indicated values.

   If a limit fails to exist, write 'd.n.e.' or use the symbols '∞' or '$-\infty$' appropriately.

   NOTE: The graph has a vertical asymptote $x = -2$ and a horizontal asymptote $y = 1$.



- $\lim_{x\to-\infty} f(x)$
- $\lim_{x\to-2^-} f(x)$
- $\lim_{x\to-2^+} f(x)$
- $\lim_{x\to\infty} f(x)$
- $\lim_{x\to 0} f(x)$
- $\lim_{x\to 2^-} f(x)$
- $f(2)$
- $\lim_{x\to 2^+} f(x)$
- List the intervals of continuity of f.

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2. Find the following limits **analytically**. Use the symbols ∞ and $-\infty$ as appropriate.

(a) $\displaystyle\lim_{x\to 3} \frac{x^2-3x}{x^2-x-6}$

(b) $\displaystyle\lim_{x\to 1} \frac{\frac{x}{x-2}+1}{x-1}$

(c) $\displaystyle\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h}$

(d) $\displaystyle\lim_{x\to 3} \frac{x^2-6x}{x^2-6x+9}$

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3. Consider the limit: $\lim_{x\to\infty} \frac{4-3x}{2x+1}$.

(a) Use the Leading Term Test to predict the value of this limit.

(b) Determine $\lim_{x\to\infty} \frac{4-3x}{2x+1}$ analytically.

4. Let $f(x) = \frac{2x-1}{x^2+4}$.

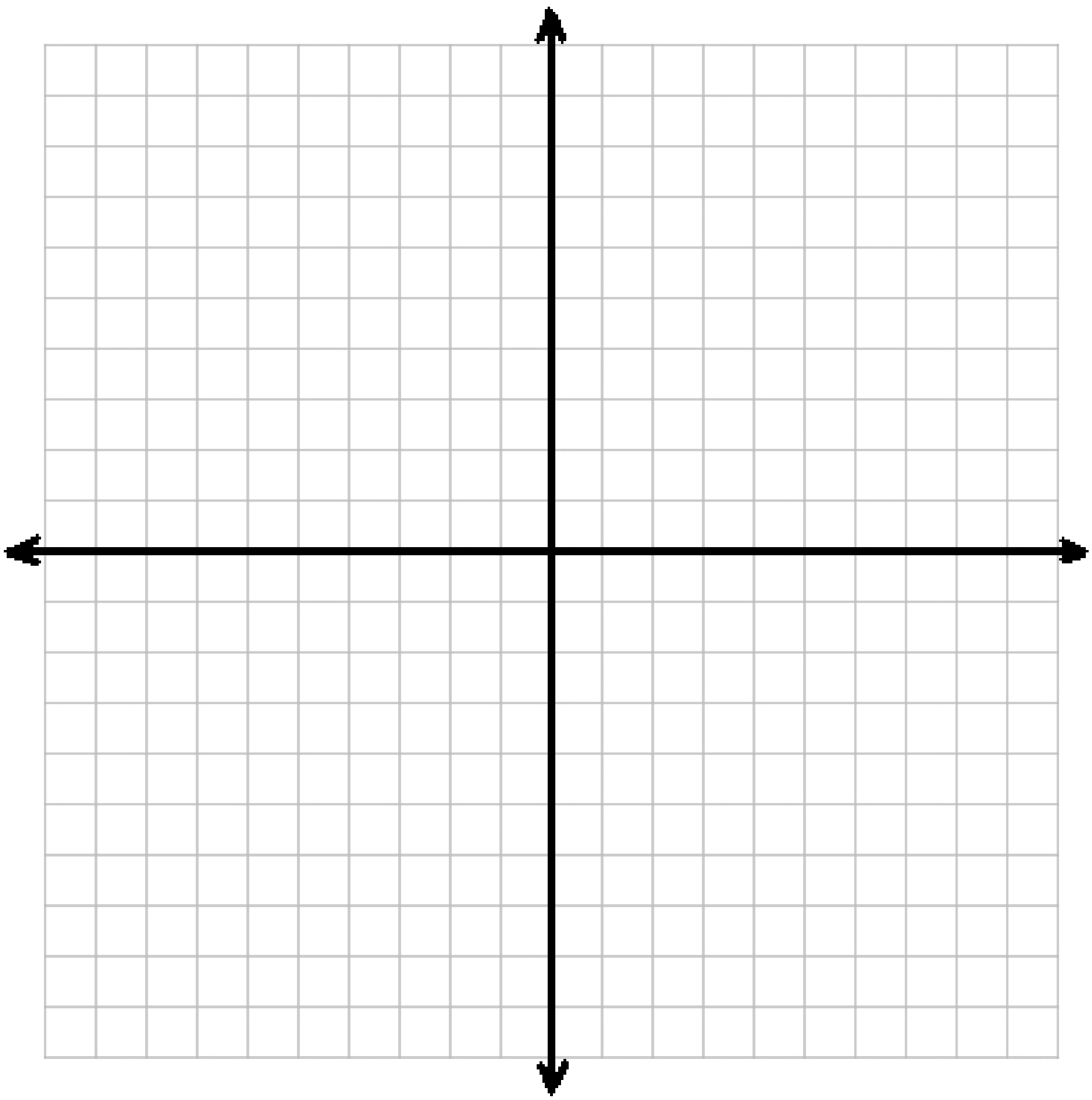
(a) Explain why the graph of f doesn't have any **vertical** asymptotes.

(b) Explain why the graph of f has a **horizontal** asymptote and find it analytically.

5. Sketch the graph of a function which satisfies all of the following criteria:

- $\lim_{x \to -\infty} f(x) = \infty$
- $\lim_{x \to 4^-} f(x) = 6$
- $\lim_{x \to 4^+} f(x) = -\infty$
- $\lim_{x \to \infty} f(x) = 0$

- The intervals of continuity of f are $(-\infty, 4]$, $(4, \infty)$.



6. Let $f(x) = \dfrac{5x - x^2}{x^2 + 3x}$

(a) Find the values of x where f is not continuous. Explain your reasoning.

(b) For each of your answers in part (a), use limits to classify the discontinuity as removable or non-removable. Explain your reasoning.

(c) Redefine $f(x)$ to remove any removable discontinuities you found in part (b). How are you changing the graph at this point?

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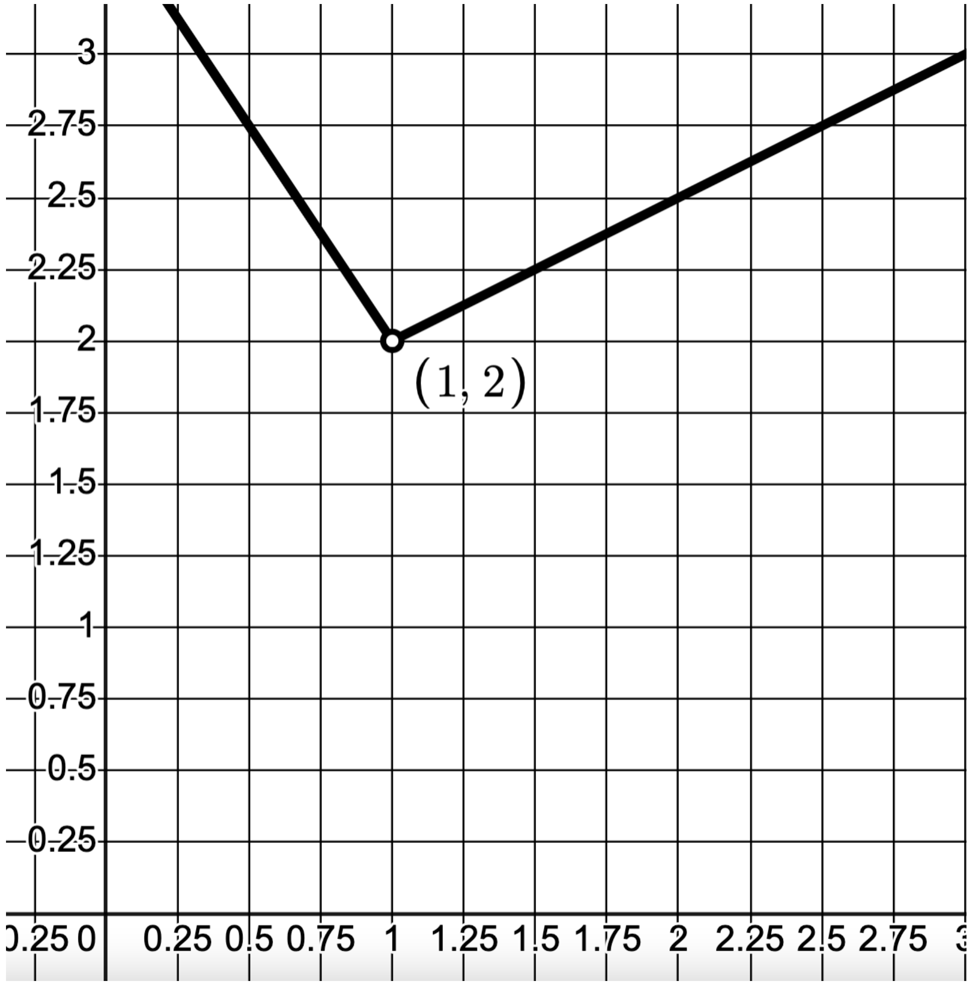
7. For the function below, $\lim_{x \to 1} f(x) = 2$.



(a) Estimate the largest $\delta > 0$ so that $|f(x) - 2| < 0.75$ if $0 < |x - 1| < \delta$.

Explain your reasoning using the graph.

8. Use the precise ($\epsilon - \delta$) definition of limit to prove $\lim_{x \to 3} (2x - 1) = 5$.

SCRATCHWORK:

FORMAL PROOF:

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9. (a) i. Use a the substitution $t = 2x$ along with the fact that $\lim_{t \to 0} \frac{\sin(t)}{t} = 1$ to find $\lim_{x \to 0} \frac{\sin(2x)}{3x}$.

ii. Use part (i) and an identity to help you find $\lim_{x \to 0} 3x \csc(2x)$.

(b) If a and b are nonzero real numbers, use the substitution $t = ax$ to show $\lim_{x \to 0} \frac{\sin(ax)}{bx} = \frac{a}{b}$.

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